

$$z^2 = w \quad (w \in \mathbb{C})$$

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$\forall w$

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$$\forall w \in \mathbb{C}, \exists z$$

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$$\forall w \in \mathbb{C}, \exists z \in \mathbb{C} \text{ s.t.}$$

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For every complex number w

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$$\forall w \in \mathbb{C}, \exists z \in \mathbb{C} \text{ s.t. } z^2 = w$$

For every complex number w , there exists

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$$\forall w \in \mathbb{C}, \exists z \in \mathbb{C} \text{ s.t. } z^2 = w$$

For every complex number w , there exists at least one complex number z

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$$\forall w \in \mathbb{C}, \exists z \in \mathbb{C} \text{ s.t. } z^2 = w$$

For every complex number w , there exists at least one complex number z such that $z^2 = w$.

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Let $z = a + bi$, $w = c + di$

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Let $z = a + bi$, $w = c + di$ ($a, b, c, d \in \mathbb{R}$)

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Let $z = a + bi$, $w = c + di$ ($a, b, c, d \in \mathbb{R}$)

$$(a + bi)^2 = c + di$$

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Let $z = a + bi$, $w = c + di$ ($a, b, c, d \in \mathbb{R}$)

$$\begin{aligned}(a + bi)^2 &= c + di \\ a^2 - b^2 + 2abi &= c + di\end{aligned}$$

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