

▶ Start

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$$a \mid b \ (a, b \in \mathbb{Z})$$

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$a \mid b (a, b \in \mathbb{Z})$  :  $a$  divides  $b$ .

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*i.e.*

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*i.e.*  $\exists k \in \mathbb{Z}$

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- $a \mid b$

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- $a \mid b, b \mid c$

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- $a \mid b, b \mid c \Rightarrow a \mid c$   
ex)  $2 \mid 6$

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- $a \mid b, b \mid c \Rightarrow a \mid c$

ex)  $2 \mid 6, 6 \mid 18$

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•  $a \mid b$

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ex)  $2 \mid 6, 6 \mid 18 \Rightarrow 2 \mid 18$

- $a \mid b, b \mid a$



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- $a \mid b, b \mid c \Rightarrow a \mid c$

ex)  $2 \mid 6, 6 \mid 18 \Rightarrow 2 \mid 18$

- $a \mid b, b \mid a \Rightarrow a = \pm b$

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- $a \mid b$

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- $a \mid b, b \mid c \Rightarrow a \mid c$

ex)  $2 \mid 6, 6 \mid 18 \Rightarrow 2 \mid 18$

- $a \mid b, b \mid a \Rightarrow a = \pm b$

- $a \mid b, a \mid c \Rightarrow a \mid (b \pm c)$

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- $a \mid b, b \mid a \Rightarrow a = \pm b$

- $a \mid b, a \mid c \Rightarrow a \mid (b \pm c)$

ex)  $3 \mid 6$

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- $a \mid b, b \mid c \Rightarrow a \mid c$

ex)  $2 \mid 6, 6 \mid 18 \Rightarrow 2 \mid 18$

- $a \mid b, b \mid a \Rightarrow a = \pm b$

- $a \mid b, a \mid c \Rightarrow a \mid (b \pm c)$

ex)  $3 \mid 6, 3 \mid 21$

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- $a \mid b, b \mid c \Rightarrow a \mid c$

ex)  $2 \mid 6, 6 \mid 18 \Rightarrow 2 \mid 18$

- $a \mid b, b \mid a \Rightarrow a = \pm b$

- $a \mid b, a \mid c \Rightarrow a \mid (b \pm c)$

ex)  $3 \mid 6, 3 \mid 21 \Rightarrow 3 \mid (6 \pm 21)$

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$a$  is a divisor of  $b$ .

$b$  is a multiple of  $a$ .

*i.e.*  $\exists k \in \mathbb{Z}$  s.t.  $b = ak$

- $a \mid b, b \mid c \Rightarrow a \mid c$  ▶ Proof

ex)  $2 \mid 6, 6 \mid 18 \Rightarrow 2 \mid 18$

- $a \mid b, b \mid a \Rightarrow a = \pm b$  ▶ Proof

- $a \mid b, a \mid c \Rightarrow a \mid (b \pm c)$  ▶ Proof

ex)  $3 \mid 6, 3 \mid 21 \Rightarrow 3 \mid (6 \pm 21)$



▶ Start

- $a \mid b, b \mid c \Rightarrow a \mid c$

▶ Start

- $a \mid b, b \mid c \Rightarrow a \mid c$

$$b = ak_1$$

▶ Start

- $a \mid b, b \mid c \Rightarrow a \mid c$

$$b = ak_1, c = bk_2$$

▶ Start

- $a \mid b, b \mid c \Rightarrow a \mid c$

$$b = ak_1, c = bk_2 \quad (k_1, k_2 \in \mathbb{Z})$$

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- $a \mid b, b \mid c \Rightarrow a \mid c$

$$b = ak_1, c = bk_2 \quad (k_1, k_2 \in \mathbb{Z})$$

$$c = (ak_1)k_2$$

▶ Start

- $a \mid b, b \mid c \Rightarrow a \mid c$

$$b = ak_1, c = bk_2 \quad (k_1, k_2 \in \mathbb{Z})$$

$$c = (ak_1)k_2 = a(k_1k_2)$$

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- $a \mid b, b \mid c \Rightarrow a \mid c$

$$b = ak_1, c = bk_2 \quad (k_1, k_2 \in \mathbb{Z})$$

$$c = (ak_1)k_2 = a(k_1k_2)$$

$$\text{Let } k_3 = k_1k_2$$

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$$c = ak_3$$



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- $a \mid b, b \mid c \Rightarrow a \mid c$

$$b = ak_1, c = bk_2 \quad (k_1, k_2 \in \mathbb{Z})$$

$$c = (ak_1)k_2 = a(k_1k_2)$$

$$\text{Let } k_3 = k_1k_2$$

$$c = ak_3$$

$$\therefore a \mid c$$

▶ Start

- $a \mid b, b \mid a \Rightarrow a = \pm b$

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- $a \mid b, b \mid a \Rightarrow a = \pm b$

$$b = ak_1, a = bk_2 \quad (k_1, k_2 \in \mathbb{Z})$$

$$a = (ak_1)k_2$$

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- $a \mid b, b \mid a \Rightarrow a = \pm b$

$$b = ak_1, a = bk_2 \quad (k_1, k_2 \in \mathbb{Z})$$

$$a = (ak_1)k_2 = ak_1k_2$$

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•  $a \mid b, b \mid a \Rightarrow a = \pm b$

$$b = ak_1, a = bk_2 \quad (k_1, k_2 \in \mathbb{Z})$$

$$a = (ak_1)k_2 = ak_1k_2$$

$$a = ak_1k_2$$



▶ Start

- $a \mid b, b \mid a \Rightarrow a = \pm b$

$$b = ak_1, a = bk_2 \quad (k_1, k_2 \in \mathbb{Z})$$

$$a = (ak_1)k_2 = ak_1k_2$$

$$a = ak_1k_2$$

$$a(k_1k_2 - 1) = 0$$

▶ Start

- $a \mid b, b \mid a \Rightarrow a = \pm b$

$$b = ak_1, a = bk_2 \quad (k_1, k_2 \in \mathbb{Z})$$

$$a = (ak_1)k_2 = ak_1k_2$$

$$a = ak_1k_2$$

$$a(k_1k_2 - 1) = 0$$

$$a = 0 \text{ or } k_1k_2 = 1$$

▶ Start

- $a \mid b, b \mid a \Rightarrow a = \pm b$

$$b = ak_1, a = bk_2 \quad (k_1, k_2 \in \mathbb{Z})$$

$$a = (ak_1)k_2 = ak_1k_2$$

$$a = ak_1k_2$$

$$a(k_1k_2 - 1) = 0$$

$$a = 0 \text{ or } k_1k_2 = 1$$

$$(a = 0 \text{ and } b = 0) \text{ or } k_1 = \pm 1$$

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- $a \mid b, b \mid a \Rightarrow a = \pm b$

$$b = ak_1, a = bk_2 \quad (k_1, k_2 \in \mathbb{Z})$$

$$a = (ak_1)k_2 = ak_1k_2$$

$$a = ak_1k_2$$

$$a(k_1k_2 - 1) = 0$$

$$a = 0 \text{ or } k_1k_2 = 1$$

$$(a = 0 \text{ and } b = 0) \text{ or } k_1 = \pm 1 (\because k_1, k_2 \in \mathbb{Z})$$

▶ Start

- $a \mid b, b \mid a \Rightarrow a = \pm b$

$$b = ak_1, a = bk_2 \quad (k_1, k_2 \in \mathbb{Z})$$

$$a = (ak_1)k_2 = ak_1k_2$$

$$a = ak_1k_2$$

$$a(k_1k_2 - 1) = 0$$

$$a = 0 \text{ or } k_1k_2 = 1$$

$$(a = 0 \text{ and } b = 0) \text{ or } k_1 = \pm 1 (\because k_1, k_2 \in \mathbb{Z})$$

$$(a = 0 \text{ and } b = 0) \text{ or } a = \pm b$$

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•  $a \mid b, b \mid a \Rightarrow a = \pm b$

$$b = ak_1, a = bk_2 \quad (k_1, k_2 \in \mathbb{Z})$$

$$a = (ak_1)k_2 = ak_1k_2$$

$$a = ak_1k_2$$

$$a(k_1k_2 - 1) = 0$$

$$a = 0 \text{ or } k_1k_2 = 1$$

$$(a = 0 \text{ and } b = 0) \text{ or } k_1 = \pm 1 (\because k_1, k_2 \in \mathbb{Z})$$

$$(a = 0 \text{ and } b = 0) \text{ or } a = \pm b$$

$$\therefore a = \pm b$$

▶ Start

- $a \mid b, a \mid c \Rightarrow a \mid (b \pm c)$

▶ Start

- $a \mid b, a \mid c \Rightarrow a \mid (b \pm c)$

$$b = ak_1$$



▶ Start

- $a \mid b, a \mid c \Rightarrow a \mid (b \pm c)$

$$b = ak_1, c = ak_2$$

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- $a \mid b, a \mid c \Rightarrow a \mid (b \pm c)$

$$b = ak_1, c = ak_2 \quad (k_1, k_2 \in \mathbb{Z})$$

▶ Start

- $a \mid b, a \mid c \Rightarrow a \mid (b \pm c)$

$$b = ak_1, c = ak_2 \quad (k_1, k_2 \in \mathbb{Z})$$

$$b \pm c = a(k_1 \pm k_2)$$

▶ Start

- $a \mid b, a \mid c \Rightarrow a \mid (b \pm c)$

$$b = ak_1, c = ak_2 \quad (k_1, k_2 \in \mathbb{Z})$$

$$b \pm c = a(k_1 \pm k_2)$$

$$\text{Let } k_3 = k_1 \pm k_2$$

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- $a \mid b, a \mid c \Rightarrow a \mid (b \pm c)$

$$b = ak_1, c = ak_2 \quad (k_1, k_2 \in \mathbb{Z})$$

$$b \pm c = a(k_1 \pm k_2)$$

$$\text{Let } k_3 = k_1 \pm k_2$$

$$b \pm c = ak_3$$

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- $a \mid b, a \mid c \Rightarrow a \mid (b \pm c)$

$$b = ak_1, c = ak_2 \quad (k_1, k_2 \in \mathbb{Z})$$

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$$\text{Let } k_3 = k_1 \pm k_2$$

$$b \pm c = ak_3$$

$$\therefore a \mid (b \pm c)$$