The Integer Division Algorithm

Start

The Integer Division Algorithm

Start

$\forall A, \forall B (\neq 0) \in \mathbb{Z}$

▸ Start

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z}$$

▸ Start

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t.$$

The Integer Division Algorithm

Start

$$\forall A, \forall B (\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R$$

The Integer Division Algorithm

Start

$$\forall A, \forall B (\neq 0) \in \mathbb{Z}, \exists ! Q, \exists ! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

The Integer Division Algorithm

Start

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

▸ Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] ▸ Proof

▸ Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] ▸ Proof [Uniqueness] ▸ Proof

The Integer Division Algorithm

▸ Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] ▸ Proof [Uniqueness] ▸ Proof

$ex) \;\; 7$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] Proof [Uniqueness] Proof

$ex) \;\; 7 \quad =$

Start

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

[Existence] Proof [Uniqueness] Proof

$$ex) \;\; 7 \;\; = \;\; 2$$

The Integer Division Algorithm

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] Proof [Uniqueness] Proof

$ex)\;\; 7 \;\; = \;\; 2 \;\; \times$

The Integer Division Algorithm

Start

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

[Existence] Proof [Uniqueness] Proof

$$ex) \; 7 \quad = \quad 2 \quad \times \quad 3$$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] Proof [Uniqueness] Proof

$$ex) \; 7 \;\; = \;\; 2 \;\; \times \;\; 3 \;\; +$$

▸ Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] ▸ Proof [Uniqueness] ▸ Proof

$ex) \; 7 \;\; = \;\; 2 \;\; \times \;\; 3 \;\; + \;\; 1$

The Integer Division Algorithm

▸ Start

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \ \ s.t. \ \ A = BQ + R, \ \ 0 \leq R < |B|$$

[Existence] ▸ Proof [Uniqueness] ▸ Proof

$$ex) \ 7 = 2 \times 3 + 1, \ 0 \leq 1 < |2|$$

The Integer Division Algorithm

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] Proof [Uniqueness] Proof

$ex)\; 7 = 2 \times 3 + 1 \;,\; 0 \leq 1 < |2|$

7

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] Proof [Uniqueness] Proof

$ex)\;\; 7 = 2 \times 3 + 1 \; , \; 0 \leq 1 < |2|$

$7 =$

The Integer Division Algorithm

▸ Start

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \ \ s.t. \ \ A = BQ + R, \ \ 0 \leq R < |B|$$

[Existence] ▸ Proof [Uniqueness] ▸ Proof

$$ex) \ 7 = 2 \times 3 + 1, \ 0 \leq 1 < |2|$$

$$7 = (-2)$$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] Proof [Uniqueness] Proof

$ex)\; 7 = 2 \times 3 + 1 \;,\; 0 \leq 1 < |2|$

$7 = (-2) \times$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] Proof [Uniqueness] Proof

$ex) \; 7 = 2 \times 3 + 1 \; , \; 0 \leq 1 < |2|$

$7 = (-2) \times (-3)$

The Integer Division Algorithm

▸ Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \ \ s.t. \ \ A = BQ + R, \ \ 0 \leq R < |B|$

[Existence] ▸ Proof [Uniqueness] ▸ Proof

$$ex) \ 7 = 2 \times 3 + 1 \ , \ 0 \leq 1 < |2|$$

$$7 = (-2) \times (-3) +$$

Start

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

[Existence] Proof [Uniqueness] Proof

$$ex)\; 7 = 2 \times 3 + 1, \quad 0 \leq 1 < |2|$$

$$7 = (-2) \times (-3) + 1$$

The Integer Division Algorithm

▸ Start

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

[Existence] ▸ Proof [Uniqueness] ▸ Proof

$$ex)\; 7 = 2 \times 3 + 1, \quad 0 \leq 1 < |2|$$

$$7 = (-2) \times (-3) + 1, \quad 0 \leq 1 < |-2|$$

▸ Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] ▸ Proof [Uniqueness] ▸ Proof

$$ex) \; 7 = 2 \times 3 + 1 \;, \; 0 \leq 1 < |2|$$

$$7 = (-2) \times (-3) + 1 \;, \; 0 \leq 1 < |-2|$$

First

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \ \ s.t. \ \ A = BQ + R, \ \ 0 \leq R < |B|$$

[Existence] Proof [Uniqueness] Proof

$$ex) \ 7 = 2 \times 3 + 1, \ 0 \leq 1 < |2|$$

$$7 = (-2) \times (-3) + 1, \ 0 \leq 1 < |-2|$$

The Integer Division Algorithm

▸ Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z}\ \ s.t.\ \ A = BQ + R,\ \ 0 \leq R < |B|$

[Existence] ▸ End

The Integer Division Algorithm

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] End

Let

The Integer Division Algorithm

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] End

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \quad \cdots \cdots (1)$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] End

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \;\; \cdots\cdots (1)$

$\exists R \in S$

The Integer Division Algorithm

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] End

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \quad \cdots\cdots (1)$

$\exists R \in S \;\; s.t.$

The Integer Division Algorithm

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] End

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \quad \cdots \cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S$

The Integer Division Algorithm

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] End

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \quad \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow$

The Integer Division Algorithm

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z}\ \ s.t.\ \ A = BQ + R,\ \ 0 \leq R < |B|$

[Existence] End

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\}\ \ \cdots\cdots(1)$

$\exists R \in S\ \ s.t.\ \ x \in S \Rightarrow R \leq x$

The Integer Division Algorithm

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] End

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \quad \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \quad (\because S \subset \mathbb{N} \cup \{0\})$

The Integer Division Algorithm

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$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] End

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \quad \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \quad (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z}$

The Integer Division Algorithm

Start

$\forall A, \forall B (\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] End

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \;\; \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \;\; (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z} \;\; s.t.$

The Integer Division Algorithm

▸ Start

$\forall A, \forall B (\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] ▸ End

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \;\; \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \;\; (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \;\; \cdots\cdots (2)$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \ \ s.t. \ \ A = BQ + R, \ \ 0 \leq R < |B|$

[Existence] End

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \quad \cdots\cdots (1)$

$\exists R \in S \ \ s.t. \ \ x \in S \Rightarrow R \leq x \quad (\because S \subset \mathbb{N} \cup \{0\})$

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$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \;\; \cdots\cdots (2)$

Assume

The Integer Division Algorithm

Start

$\forall A, \forall B (\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] End

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$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \;\; (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \;\; \cdots\cdots (2)$

Assume $R \geq |B|$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] End

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \;\; \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \quad (\because S \subset \mathbb{N} \cup \{0\})$

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Assume $R \geq |B|$

$A - BQ \geq |B|$

The Integer Division Algorithm

▸ Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] ▸ End

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \;\; \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \;\; (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \;\; \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B| \;\; , \; A - BQ - |B| \geq 0$

The Integer Division Algorithm

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z}\ \ s.t.\ \ A = BQ + R,\ \ 0 \leq R < |B|$

[Existence] End

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \quad \cdots\cdots (1)$

$\exists R \in S\ \ s.t.\ \ x \in S \Rightarrow R \leq x \quad (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z}\ \ s.t.\ \ R = A - BQ \quad \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B|\ \ ,\ A - BQ - |B| \geq 0$

$A - B\left(Q + \dfrac{|B|}{B}\right)$

The Integer Division Algorithm

▸ Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] ▸ End

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \;\; \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \;\; (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \;\; \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B| \;\; , \; A - BQ - |B| \geq 0$

$$A - B\left(Q + \frac{|B|}{B}\right) =$$

The Integer Division Algorithm

▸ Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] ▸ End

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \;\; \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \;\; (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \;\; \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B| \;\; , \; A - BQ - |B| \geq 0$

$$A - B\left(Q + \frac{|B|}{B}\right) = A - BQ - |B|$$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] End

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \quad \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \quad (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \quad \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B| \;\; , \; A - BQ - |B| \geq 0$

$A - B\left(Q + \frac{|B|}{B}\right) = A - BQ - |B| <$

The Integer Division Algorithm

▸ Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] ▸ End

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \quad \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \quad (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \quad \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B| \;\; , \; A - BQ - |B| \geq 0$

$$A - B\left(Q + \frac{|B|}{B}\right) = A - BQ - |B| < A - BQ$$

The Integer Division Algorithm

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] End

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \quad \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \quad (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \quad \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B| \;\; , \; A - BQ - |B| \geq 0$

$$A - B\left(Q + \frac{|B|}{B}\right) = A - BQ - |B| < A - BQ \quad \therefore contradiction$$

▸ Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] ▸ End

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \;\; \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \;\; (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \;\; \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B| \;\; , \; A - BQ - |B| \geq 0$

$A - B\left(Q + \frac{|B|}{B}\right) = A - BQ - |B| < A - BQ \quad \therefore contradiction$

$\therefore R < |B| \;\; \cdots\cdots (3)$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] End

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \;\; \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \quad (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \;\; \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B| \;\; , \; A - BQ - |B| \geq 0$

$A - B\left(Q + \frac{|B|}{B}\right) = A - BQ - |B| < A - BQ \quad \therefore contradiction$

$\therefore R < |B| \;\; \cdots\cdots (3)$

By (1), (2), (3)

$\therefore$

Start

$\forall A, \forall B (\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] End

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \quad \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \quad (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \quad \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B| \;\; , \; A - BQ - |B| \geq 0$

$A - B\left(Q + \frac{|B|}{B}\right) = A - BQ - |B| < A - BQ \quad \therefore contradiction$

$\therefore R < |B| \quad \cdots\cdots (3)$

By (1), (2), (3)

$\therefore \exists Q, \exists R \in \mathbb{Z}$

The Integer Division Algorithm

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z}\ \ s.t.\ \ A = BQ + R, \ \ 0 \leq R < |B|$

[Existence] End

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \ \ \cdots\cdots (1)$

$\exists R \in S\ \ s.t.\ \ x \in S \Rightarrow R \leq x \ \ (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z}\ \ s.t.\ \ R = A - BQ \ \ \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B|\ \ ,\ A - BQ - |B| \geq 0$

$$A - B\left(Q + \frac{|B|}{B}\right) = A - BQ - |B| < A - BQ \quad \therefore contradiction$$

$\therefore R < |B| \ \ \cdots\cdots (3)$

By (1), (2), (3)

$\therefore \exists Q, \exists R \in \mathbb{Z}\ \ s.t.$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z}\ \ s.t.\ \ A = BQ + R,\ \ 0 \leq R < |B|$

[Existence] End

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\}\ \ \cdots\cdots (1)$

$\exists R \in S\ \ s.t.\ \ x \in S \Rightarrow R \leq x\ \ (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z}\ \ s.t.\ \ R = A - BQ\ \ \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B|\ \ ,\ A - BQ - |B| \geq 0$

$A - B\left(Q + \frac{|B|}{B}\right) = A - BQ - |B| < A - BQ\ \ \ \therefore contradiction$

$\therefore R < |B|\ \ \cdots\cdots (3)$

By (1), (2), (3)

$\therefore \exists Q, \exists R \in \mathbb{Z}\ \ s.t.\ \ A = BQ + R,$

▸ Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \ \ s.t. \ \ A = BQ + R, \ \ 0 \leq R < |B|$

[Existence] ▸ End

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \ \ \cdots\cdots (1)$

$\exists R \in S \ \ s.t. \ \ x \in S \Rightarrow R \leq x \ \ (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z} \ \ s.t. \ \ R = A - BQ \ \ \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B| \ , \ A - BQ - |B| \geq 0$

$A - B\left(Q + \frac{|B|}{B}\right) = A - BQ - |B| < A - BQ \quad \therefore contradiction$

$\therefore R < |B| \ \ \cdots\cdots (3)$

By (1), (2), (3)

$\therefore \exists Q, \exists R \in \mathbb{Z} \ \ s.t. \ \ A = BQ + R, \ \ 0 \leq R < |B|$

The Integer Division Algorithm

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] First

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \;\; \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \;\; (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \;\; \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B| \;\; , \; A - BQ - |B| \geq 0$

$A - B\left(Q + \frac{|B|}{B}\right) = A - BQ - |B| < A - BQ \quad \therefore contradiction$

$\therefore R < |B| \;\; \cdots\cdots (3)$

By (1), (2), (3)

$\therefore \exists Q, \exists R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Uniquness] End

The Integer Division Algorithm

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Uniqupness] End

$Let\ A = BQ_1 + R_1, \;\; 0 \leq R_1 < |B|$

▸ Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Uniqupness] ▸ End

$Let\ A = BQ_1 + R_1, \;\; 0 \leq R_1 < |B|$

$Let\ A = BQ_2 + R_2, \;\; 0 \leq R_2 < |B|$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Uniquness] End

$Let \; A = BQ_1 + R_1, \;\; 0 \leq R_1 < |B|$

$Let \; A = BQ_2 + R_2, \;\; 0 \leq R_2 < |B|$

$-|B| < R_2 - R_1 < |B|$

The Integer Division Algorithm

Start

$\forall A, \forall B (\neq 0) \in \mathbb{Z}, \exists ! Q, \exists ! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Uniqueness] End

$Let\ A = BQ_1 + R_1, \;\; 0 \leq R_1 < |B|$

$Let\ A = BQ_2 + R_2, \;\; 0 \leq R_2 < |B|$

$-|B| < R_2 - R_1 < |B| \;\; , \;\; |R_2 - R_1| < |B| \;\; \cdots\cdots (1)$

The Integer Division Algorithm

Start

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

[Uniqupness] End

$$Let \; A = BQ_1 + R_1, \;\; 0 \leq R_1 < |B|$$

$$Let \; A = BQ_2 + R_2, \;\; 0 \leq R_2 < |B|$$

$$-|B| < R_2 - R_1 < |B| \;\; , \;\; |R_2 - R_1| < |B| \;\; \cdots\cdots (1)$$

$$BQ_1 + R_1 = BQ_2 + R_2$$

Start

$\forall A, \forall B (\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \ \ s.t. \ \ A = BQ + R, \ \ 0 \leq R < |B|$

[Uniqus] End

$Let\ A = BQ_1 + R_1, \ \ 0 \leq R_1 < |B|$

$Let\ A = BQ_2 + R_2, \ \ 0 \leq R_2 < |B|$

$-|B| < R_2 - R_1 < |B| \ \ , \ \ |R_2 - R_1| < |B| \ \ \cdots\cdots (1)$

$BQ_1 + R_1 = BQ_2 + R_2 \ \ , \ \ B(Q_1 - Q_2) = R_2 - R_1 \ \ \cdots\cdots (2)$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Uniquness] End

$Let\ A = BQ_1 + R_1, \;\; 0 \leq R_1 < |B|$

$Let\ A = BQ_2 + R_2, \;\; 0 \leq R_2 < |B|$

$-|B| < R_2 - R_1 < |B| \;\; , \;\; |R_2 - R_1| < |B| \;\; \cdots\cdots (1)$

$BQ_1 + R_1 = BQ_2 + R_2 \;\; , \;\; B(Q_1 - Q_2) = R_2 - R_1 \;\; \cdots\cdots (2)$

$by\ (1), (2)$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Uniqupness] End

$Let \; A = BQ_1 + R_1, \;\; 0 \leq R_1 < |B|$

$Let \; A = BQ_2 + R_2, \;\; 0 \leq R_2 < |B|$

$-|B| < R_2 - R_1 < |B| \;\; , \;\; |R_2 - R_1| < |B| \;\; \cdots\cdots (1)$

$BQ_1 + R_1 = BQ_2 + R_2 \;\; , \;\; B(Q_1 - Q_2) = R_2 - R_1 \;\; \cdots\cdots (2)$

$by \; (1), (2)$

$|B(Q_1 - Q_2)| = |R_2 - R_1| < |B|$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Uniquness] End

$Let \; A = BQ_1 + R_1, \;\; 0 \leq R_1 < |B|$

$Let \; A = BQ_2 + R_2, \;\; 0 \leq R_2 < |B|$

$-|B| < R_2 - R_1 < |B| \;\; , \;\; |R_2 - R_1| < |B| \;\; \cdots\cdots (1)$

$BQ_1 + R_1 = BQ_2 + R_2 \;\; , \;\; B(Q_1 - Q_2) = R_2 - R_1 \;\; \cdots\cdots (2)$

$by \; (1), (2)$

$|B(Q_1 - Q_2)| = |R_2 - R_1| < |B|$

$|B||Q_1 - Q_2| < |B|$

The Integer Division Algorithm

▸ Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z}\ \ s.t.\ \ A = BQ + R,\ \ 0 \leq R < |B|$

[Uniquness] ▸ End

$Let\ A = BQ_1 + R_1,\ \ 0 \leq R_1 < |B|$

$Let\ A = BQ_2 + R_2,\ \ 0 \leq R_2 < |B|$

$-|B| < R_2 - R_1 < |B|\ \ ,\ \ |R_2 - R_1| < |B|\ \ \cdots\cdots(1)$

$BQ_1 + R_1 = BQ_2 + R_2\ \ ,\ \ B(Q_1 - Q_2) = R_2 - R_1\ \ \cdots\cdots(2)$

$by\ (1), (2)$

$|B(Q_1 - Q_2)| = |R_2 - R_1| < |B|$

$|B||Q_1 - Q_2| < |B|\ \ ,\ \ |Q_1 - Q_2| < 1$

The Integer Division Algorithm

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Uniqueness] End

$Let\ A = BQ_1 + R_1, \;\; 0 \leq R_1 < |B|$

$Let\ A = BQ_2 + R_2, \;\; 0 \leq R_2 < |B|$

$-|B| < R_2 - R_1 < |B| \;\; , \;\; |R_2 - R_1| < |B| \;\; \cdots\cdots (1)$

$BQ_1 + R_1 = BQ_2 + R_2 \;\; , \;\; B(Q_1 - Q_2) = R_2 - R_1 \;\; \cdots\cdots (2)$

$by\ (1), (2)$

$|B(Q_1 - Q_2)| = |R_2 - R_1| < |B|$

$|B||Q_1 - Q_2| < |B| \;\; , \;\; |Q_1 - Q_2| < 1$

$|Q_1 - Q_2| = 0$

The Integer Division Algorithm

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Uniquness] End

$Let\ A = BQ_1 + R_1, \;\; 0 \leq R_1 < |B|$

$Let\ A = BQ_2 + R_2, \;\; 0 \leq R_2 < |B|$

$-|B| < R_2 - R_1 < |B| \;\; , \;\; |R_2 - R_1| < |B| \;\; \cdots\cdots (1)$

$BQ_1 + R_1 = BQ_2 + R_2 \;\; , \;\; B(Q_1 - Q_2) = R_2 - R_1 \;\; \cdots\cdots (2)$

$by\ (1), (2)$

$|B(Q_1 - Q_2)| = |R_2 - R_1| < |B|$

$|B||Q_1 - Q_2| < |B| \;\; , \;\; |Q_1 - Q_2| < 1$

$|Q_1 - Q_2| = 0 \;\; (\because Q_1, Q_2 \in \mathbb{Z})$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z}\ \ s.t.\ \ A = BQ + R,\ \ 0 \leq R < |B|$

[Uniquness] End

$Let\ A = BQ_1 + R_1,\ \ 0 \leq R_1 < |B|$

$Let\ A = BQ_2 + R_2,\ \ 0 \leq R_2 < |B|$

$-|B| < R_2 - R_1 < |B|\ \ ,\ \ |R_2 - R_1| < |B|\ \ \cdots\cdots (1)$

$BQ_1 + R_1 = BQ_2 + R_2\ \ ,\ \ B(Q_1 - Q_2) = R_2 - R_1\ \ \cdots\cdots (2)$

$by\ (1), (2)$

$|B(Q_1 - Q_2)| = |R_2 - R_1| < |B|$

$|B||Q_1 - Q_2| < |B|\ \ ,\ \ |Q_1 - Q_2| < 1$

$|Q_1 - Q_2| = 0\ \ (\because Q_1, Q_2 \in \mathbb{Z})$

$\therefore$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Uniqness] End

$Let \; A = BQ_1 + R_1, \;\; 0 \leq R_1 < |B|$

$Let \; A = BQ_2 + R_2, \;\; 0 \leq R_2 < |B|$

$-|B| < R_2 - R_1 < |B| \;\; , \;\; |R_2 - R_1| < |B| \;\; \cdots\cdots (1)$

$BQ_1 + R_1 = BQ_2 + R_2 \;\; , \;\; B(Q_1 - Q_2) = R_2 - R_1 \;\; \cdots\cdots (2)$

$by \; (1), (2)$

$|B(Q_1 - Q_2)| = |R_2 - R_1| < |B|$

$|B||Q_1 - Q_2| < |B| \;\; , \;\; |Q_1 - Q_2| < 1$

$|Q_1 - Q_2| = 0 \;\; (\because Q_1, Q_2 \in \mathbb{Z})$

$\therefore \;\; Q_1 = Q_2$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \ \ s.t. \ \ A = BQ + R, \ \ 0 \leq R < |B|$

[Uniquness] End

$Let \ A = BQ_1 + R_1, \ \ 0 \leq R_1 < |B|$

$Let \ A = BQ_2 + R_2, \ \ 0 \leq R_2 < |B|$

$-|B| < R_2 - R_1 < |B| \ \ , \ \ |R_2 - R_1| < |B| \ \ \cdots\cdots (1)$

$BQ_1 + R_1 = BQ_2 + R_2 \ \ , \ \ B(Q_1 - Q_2) = R_2 - R_1 \ \ \cdots\cdots (2)$

$by \ (1), (2)$

$|B(Q_1 - Q_2)| = |R_2 - R_1| < |B|$

$|B||Q_1 - Q_2| < |B| \ \ , \ \ |Q_1 - Q_2| < 1$

$|Q_1 - Q_2| = 0 \ \ (\because Q_1, Q_2 \in \mathbb{Z})$

$\therefore \ Q_1 = Q_2 \ \ , \ \ R_1 = R_2$

The Integer Division Algorithm

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z}\ \ s.t.\ \ A = BQ + R,\ \ 0 \leq R < |B|$

[Uniquness] First

$Let\ A = BQ_1 + R_1,\ \ 0 \leq R_1 < |B|$

$Let\ A = BQ_2 + R_2,\ \ 0 \leq R_2 < |B|$

$-|B| < R_2 - R_1 < |B|\ \ ,\ \ |R_2 - R_1| < |B|\ \ \cdots\cdots(1)$

$BQ_1 + R_1 = BQ_2 + R_2\ \ ,\ \ B(Q_1 - Q_2) = R_2 - R_1\ \ \cdots\cdots(2)$

$by\ (1),(2)$

$|B(Q_1 - Q_2)| = |R_2 - R_1| < |B|$

$|B||Q_1 - Q_2| < |B|\ \ ,\ \ |Q_1 - Q_2| < 1$

$|Q_1 - Q_2| = 0\ \ (\because Q_1, Q_2 \in \mathbb{Z})$

$\therefore\ Q_1 = Q_2\ \ ,\ \ R_1 = R_2$