

# The Complex Conjugate

▶ Start

▶ Start

## Complex conjugates

▶ Start

## Complex conjugates

a pair of complex numbers,

▶ Start

## Complex conjugates

a pair of complex numbers, both having the same real part,

▶ Start

## Complex conjugates

a pair of complex numbers, both having the same real part, but with imaginary parts of

▶ Start

## Complex conjugates

a pair of complex numbers, both having the same real part, but with imaginary parts of equal magnitude

▶ Start

## Complex conjugates

a pair of complex numbers, both having the same real part, but with imaginary parts of equal magnitude and opposite signs.

# The Complex Conjugate

▶ Start

## Complex conjugates

a pair of complex numbers, both having the same real part, but with imaginary parts of equal magnitude and opposite signs.

*ex)*  $1 + 2i$ ,  $1 - 2i$  are complex conjugates.



# The Complex Conjugate

▶ Start

## Complex conjugates

a pair of complex numbers, both having the same real part, but with imaginary parts of equal magnitude and opposite signs.

ex)  $1 + 2i$ ,  $1 - 2i$  are complex conjugates.

$$\overline{a + bi} = a - bi$$

# The Complex Conjugate

▶ Start

## Complex conjugates

a pair of complex numbers, both having the same real part, but with imaginary parts of equal magnitude and opposite signs.

ex)  $1 + 2i, 1 - 2i$  are complex conjugates.

$$\overline{a + bi} = a - bi \quad (a, b \in \mathbb{R})$$

# The Complex Conjugate

▶ Start

## Complex conjugates

a pair of complex numbers, both having the same real part, but with imaginary parts of equal magnitude and opposite signs.

ex)  $1 + 2i, 1 - 2i$  are complex conjugates.

$$\overline{a + bi} = a - bi \quad (a, b \in \mathbb{R})$$

The conjugate of the complex number  $a + bi$

▶ Start

## Complex conjugates

a pair of complex numbers, both having the same real part, but with imaginary parts of equal magnitude and opposite signs.

ex)  $1 + 2i, 1 - 2i$  are complex conjugates.

$$\overline{a + bi} = a - bi \quad (a, b \in \mathbb{R})$$

The conjugate of the complex number  $a + bi$

- $\overline{z \pm w}$

▶ Start

## Complex conjugates

a pair of complex numbers, both having the same real part, but with imaginary parts of equal magnitude and opposite signs.

ex)  $1 + 2i, 1 - 2i$  are complex conjugates.

$$\overline{a + bi} = a - bi \quad (a, b \in \mathbb{R})$$

The conjugate of the complex number  $a + bi$

- $\overline{z \pm w} = \bar{z} \pm \bar{w}$

▶ Start

## Complex conjugates

a pair of complex numbers, both having the same real part, but with imaginary parts of equal magnitude and opposite signs.

ex)  $1 + 2i$ ,  $1 - 2i$  are complex conjugates.

$$\overline{a + bi} = a - bi \quad (a, b \in \mathbb{R})$$

The conjugate of the complex number  $a + bi$

- $\overline{z \pm w} = \bar{z} \pm \bar{w}$
- $\overline{z \cdot w}$

▶ Start

## Complex conjugates

a pair of complex numbers, both having the same real part, but with imaginary parts of equal magnitude and opposite signs.

ex)  $1 + 2i$ ,  $1 - 2i$  are complex conjugates.

$$\overline{a + bi} = a - bi \quad (a, b \in \mathbb{R})$$

The conjugate of the complex number  $a + bi$

- $\overline{z \pm w} = \bar{z} \pm \bar{w}$
- $\overline{z \cdot w} = \bar{z} \cdot \bar{w}$

▶ Start

## Complex conjugates

a pair of complex numbers, both having the same real part, but with imaginary parts of equal magnitude and opposite signs.

ex)  $1 + 2i$ ,  $1 - 2i$  are complex conjugates.

$$\overline{a + bi} = a - bi \quad (a, b \in \mathbb{R})$$

The conjugate of the complex number  $a + bi$

- $\overline{z \pm w} = \bar{z} \pm \bar{w}$
- $\overline{z \cdot w} = \bar{z} \cdot \bar{w}$
- $\overline{\left(\frac{z}{w}\right)}$



▶ Start

## Complex conjugates

a pair of complex numbers, both having the same real part, but with imaginary parts of equal magnitude and opposite signs.

ex)  $1 + 2i$ ,  $1 - 2i$  are complex conjugates.

$$\overline{a + bi} = a - bi \quad (a, b \in \mathbb{R})$$

The conjugate of the complex number  $a + bi$

- $\overline{z \pm w} = \bar{z} \pm \bar{w}$
- $\overline{z \cdot w} = \bar{z} \cdot \bar{w}$
- $\overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$

▶ Start

## Complex conjugates

a pair of complex numbers, both having the same real part, but with imaginary parts of equal magnitude and opposite signs.

ex)  $1 + 2i$ ,  $1 - 2i$  are complex conjugates.

$$\overline{a + bi} = a - bi \quad (a, b \in \mathbb{R})$$

The conjugate of the complex number  $a + bi$

- $\overline{z \pm w} = \bar{z} \pm \bar{w}$
- $\overline{z \cdot w} = \bar{z} \cdot \bar{w}$
- $\overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$
- $\bar{\bar{z}} = z$

▶ Start

## Complex conjugates

a pair of complex numbers, both having the same real part, but with imaginary parts of equal magnitude and opposite signs.

ex)  $1 + 2i$ ,  $1 - 2i$  are complex conjugates.

$$\overline{a + bi} = a - bi \quad (a, b \in \mathbb{R})$$

The conjugate of the complex number  $a + bi$

- $\overline{z \pm w} = \bar{z} \pm \bar{w}$
- $\overline{z \cdot w} = \bar{z} \cdot \bar{w}$
- $\overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$
- $\bar{z} = z \Rightarrow z$  is a real number.

▶ Start

## Complex conjugates

a pair of complex numbers, both having the same real part, but with imaginary parts of equal magnitude and opposite signs.

ex)  $1 + 2i$ ,  $1 - 2i$  are complex conjugates.

$$\overline{a + bi} = a - bi \quad (a, b \in \mathbb{R})$$

The conjugate of the complex number  $a + bi$

- $\overline{z \pm w} = \bar{z} \pm \bar{w}$
- $\overline{z \cdot w} = \bar{z} \cdot \bar{w}$
- $\overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$
- $\bar{z} = z \Rightarrow z$  is a real number.
- $\bar{\bar{z}} = z$

▶ Start

## Complex conjugates

a pair of complex numbers, both having the same real part, but with imaginary parts of equal magnitude and opposite signs.

ex)  $1 + 2i$ ,  $1 - 2i$  are complex conjugates.

$$\overline{a + bi} = a - bi \quad (a, b \in \mathbb{R})$$

The conjugate of the complex number  $a + bi$

- $\overline{z \pm w} = \bar{z} \pm \bar{w}$
- $\overline{z \cdot w} = \bar{z} \cdot \bar{w}$
- $\overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$
- $\bar{z} = z \Rightarrow z$  is a real number.
- $\bar{z} = -z \Rightarrow z$  is an imaginary number.

▶ First

## Complex conjugates

a pair of complex numbers, both having the same real part, but with imaginary parts of equal magnitude and opposite signs.

ex)  $1 + 2i, 1 - 2i$  are complex conjugates.

$$\overline{a + bi} = a - bi \quad (a, b \in \mathbb{R})$$

The conjugate of the complex number  $a + bi$

- $\overline{z \pm w} = \bar{z} \pm \bar{w}$  ▶ Proof
- $\overline{z \cdot w} = \bar{z} \cdot \bar{w}$  ▶ Proof
- $\overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$  ▶ Proof
- $\bar{z} = z \Rightarrow z$  is a real number. ▶ Proof
- $\bar{z} = -z \Rightarrow z$  is an imaginary number. ▶ Proof

# The Complex Conjugate

▶ Start

- $\overline{z \pm w} = \bar{z} \pm \bar{w}$

# The Complex Conjugate

▶ Start

- $\overline{z \pm w} = \bar{z} \pm \bar{w}$

Let  $z = a_1 + b_1i$ ,  $w = a_2 + b_2i$  ( $a_1, a_2, b_1, b_2 \in \mathbb{R}$ )



# The Complex Conjugate

▶ Start

- $\overline{z \pm w} = \bar{z} \pm \bar{w}$

Let  $z = a_1 + b_1i$ ,  $w = a_2 + b_2i$  ( $a_1, a_2, b_1, b_2 \in \mathbb{R}$ )

$$\overline{z \pm w} = \overline{(a_1 + b_1i) \pm (a_2 + b_2i)} \text{ (Double signs in same order)}$$

# The Complex Conjugate

▶ Start

- $\overline{z \pm w} = \bar{z} \pm \bar{w}$

Let  $z = a_1 + b_1i$ ,  $w = a_2 + b_2i$  ( $a_1, a_2, b_1, b_2 \in \mathbb{R}$ )

$$\begin{aligned}\overline{z \pm w} &= \overline{(a_1 + b_1i) \pm (a_2 + b_2i)} \text{ (Double signs in same order)} \\ &= \overline{(a_1 \pm a_2) + (b_1 \pm b_2)i}\end{aligned}$$

# The Complex Conjugate

▶ Start

- $\overline{z \pm w} = \bar{z} \pm \bar{w}$

Let  $z = a_1 + b_1i$ ,  $w = a_2 + b_2i$  ( $a_1, a_2, b_1, b_2 \in \mathbb{R}$ )

$$\begin{aligned}\overline{z \pm w} &= \overline{(a_1 + b_1i) \pm (a_2 + b_2i)} \text{ (Double signs in same order)} \\ &= \overline{(a_1 \pm a_2) + (b_1 \pm b_2)i} \\ &= (a_1 \pm a_2) - (b_1 \pm b_2)i\end{aligned}$$

# The Complex Conjugate

▶ Start

- $\overline{z \pm w} = \bar{z} \pm \bar{w}$

Let  $z = a_1 + b_1i$ ,  $w = a_2 + b_2i$  ( $a_1, a_2, b_1, b_2 \in \mathbb{R}$ )

$$\begin{aligned}\overline{z \pm w} &= \overline{(a_1 + b_1i) \pm (a_2 + b_2i)} \text{ (Double signs in same order)} \\ &= \overline{(a_1 \pm a_2) + (b_1 \pm b_2)i} \\ &= (a_1 \pm a_2) - (b_1 \pm b_2)i \\ &= (a_1 - b_1i) \pm (a_2 - b_2i)\end{aligned}$$

# The Complex Conjugate

▶ Start

- $\overline{z \pm w} = \bar{z} \pm \bar{w}$

Let  $z = a_1 + b_1i$ ,  $w = a_2 + b_2i$  ( $a_1, a_2, b_1, b_2 \in \mathbb{R}$ )

$$\begin{aligned}\overline{z \pm w} &= \overline{(a_1 + b_1i) \pm (a_2 + b_2i)} \text{ (Double signs in same order)} \\ &= \overline{(a_1 \pm a_2) + (b_1 \pm b_2)i} \\ &= (a_1 \pm a_2) - (b_1 \pm b_2)i \\ &= \overline{(a_1 - b_1i) \pm (a_2 - b_2i)} \\ &= \overline{a_1 + b_1i} \pm \overline{a_2 + b_2i}\end{aligned}$$

# The Complex Conjugate

▶ Start

- $\overline{z \pm w} = \bar{z} \pm \bar{w}$

Let  $z = a_1 + b_1i$ ,  $w = a_2 + b_2i$  ( $a_1, a_2, b_1, b_2 \in \mathbb{R}$ )

$$\begin{aligned}\overline{z \pm w} &= \overline{(a_1 + b_1i) \pm (a_2 + b_2i)} \text{ (Double signs in same order)} \\ &= \overline{(a_1 \pm a_2) + (b_1 \pm b_2)i} \\ &= \overline{(a_1 \pm a_2) - (b_1 \pm b_2)i} \\ &= \overline{(a_1 - b_1i) \pm (a_2 - b_2i)} \\ &= \overline{a_1 + b_1i \pm a_2 + b_2i} \\ &= \bar{z} \pm \bar{w}\end{aligned}$$

# The Complex Conjugate

▶ Start

- $\overline{z \cdot w} = \bar{z} \cdot \bar{w}$

# The Complex Conjugate

▶ Start

- $\overline{z \cdot w} = \bar{z} \cdot \bar{w}$

Let  $z = a_1 + b_1i$ ,  $w = a_2 + b_2i$  ( $a_1, a_2, b_1, b_2 \in \mathbb{R}$ )



# The Complex Conjugate

▶ Start

- $\overline{z \cdot w} = \bar{z} \cdot \bar{w}$

Let  $z = a_1 + b_1i$ ,  $w = a_2 + b_2i$  ( $a_1, a_2, b_1, b_2 \in \mathbb{R}$ )

$$\overline{z \cdot w} = \overline{(a_1 + b_1i) \cdot (a_2 + b_2i)}$$

# The Complex Conjugate

▶ Start

- $\overline{z \cdot w} = \bar{z} \cdot \bar{w}$

Let  $z = a_1 + b_1i$ ,  $w = a_2 + b_2i$  ( $a_1, a_2, b_1, b_2 \in \mathbb{R}$ )

$$\begin{aligned}\overline{z \cdot w} &= \overline{(a_1 + b_1i) \cdot (a_2 + b_2i)} \\ &= \overline{(a_1a_2 - b_1b_2) + (a_1b_2 + a_2b_1)i}\end{aligned}$$

# The Complex Conjugate

▶ Start

- $\overline{z \cdot w} = \bar{z} \cdot \bar{w}$

Let  $z = a_1 + b_1i$ ,  $w = a_2 + b_2i$  ( $a_1, a_2, b_1, b_2 \in \mathbb{R}$ )

$$\begin{aligned}\overline{z \cdot w} &= \overline{(a_1 + b_1i) \cdot (a_2 + b_2i)} \\ &= \overline{(a_1a_2 - b_1b_2) + (a_1b_2 + a_2b_1)i} \\ &= (a_1a_2 - b_1b_2) - (a_1b_2 + a_2b_1)i\end{aligned}$$

# The Complex Conjugate

▶ Start

- $\overline{z \cdot w} = \bar{z} \cdot \bar{w}$

Let  $z = a_1 + b_1i$ ,  $w = a_2 + b_2i$  ( $a_1, a_2, b_1, b_2 \in \mathbb{R}$ )

$$\begin{aligned}\overline{z \cdot w} &= \overline{(a_1 + b_1i) \cdot (a_2 + b_2i)} \\ &= \overline{(a_1a_2 - b_1b_2) + (a_1b_2 + a_2b_1)i} \\ &= (a_1a_2 - b_1b_2) - (a_1b_2 + a_2b_1)i \\ &= (a_1 - b_1i) \cdot (a_2 - b_2i)\end{aligned}$$

# The Complex Conjugate

▶ Start

- $\overline{z \cdot w} = \bar{z} \cdot \bar{w}$

Let  $z = a_1 + b_1i$ ,  $w = a_2 + b_2i$  ( $a_1, a_2, b_1, b_2 \in \mathbb{R}$ )

$$\begin{aligned}\overline{z \cdot w} &= \overline{(a_1 + b_1i) \cdot (a_2 + b_2i)} \\ &= \overline{(a_1a_2 - b_1b_2) + (a_1b_2 + a_2b_1)i} \\ &= (a_1a_2 - b_1b_2) - (a_1b_2 + a_2b_1)i \\ &= \overline{(a_1 - b_1i) \cdot (a_2 - b_2i)} \\ &= \overline{a_1 + b_1i} \cdot \overline{a_2 + b_2i}\end{aligned}$$

# The Complex Conjugate

▶ Start

- $\overline{z \cdot w} = \bar{z} \cdot \bar{w}$

Let  $z = a_1 + b_1i$ ,  $w = a_2 + b_2i$  ( $a_1, a_2, b_1, b_2 \in \mathbb{R}$ )

$$\begin{aligned}\overline{z \cdot w} &= \overline{(a_1 + b_1i) \cdot (a_2 + b_2i)} \\ &= \overline{(a_1a_2 - b_1b_2) + (a_1b_2 + a_2b_1)i} \\ &= (a_1a_2 - b_1b_2) - (a_1b_2 + a_2b_1)i \\ &= \overline{(a_1 - b_1i) \cdot (a_2 - b_2i)} \\ &= \overline{a_1 + b_1i} \cdot \overline{a_2 + b_2i} \\ &= \bar{z} \cdot \bar{w}\end{aligned}$$

# The Complex Conjugate

▶ Start

- $\overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$

# The Complex Conjugate

▶ Start

- $\overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$

Let  $z = a_1 + b_1i$ ,  $w = a_2 + b_2i$  ( $a_1, a_2, b_1, b_2 \in \mathbb{R}$ )



# The Complex Conjugate

▶ Start

- $\overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$

Let  $z = a_1 + b_1i$ ,  $w = a_2 + b_2i$  ( $a_1, a_2, b_1, b_2 \in \mathbb{R}$ )

$$\overline{\left(\frac{z}{w}\right)}$$

# The Complex Conjugate

▶ Start

- $\overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$

Let  $z = a_1 + b_1i$ ,  $w = a_2 + b_2i$  ( $a_1, a_2, b_1, b_2 \in \mathbb{R}$ )

$$\overline{\left(\frac{z}{w}\right)} = \frac{\overline{a_1 + b_1i}}{\overline{a_2 + b_2i}}$$

# The Complex Conjugate

▶ Start

- $\overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$

Let  $z = a_1 + b_1i$ ,  $w = a_2 + b_2i$  ( $a_1, a_2, b_1, b_2 \in \mathbb{R}$ )

$$\overline{\left(\frac{z}{w}\right)} = \frac{\overline{a_1 + b_1i}}{a_2 + b_2i} = \frac{a_1a_2 - b_1b_2}{a_2^2 + b_2^2} + \frac{-a_1b_2 + a_2b_1}{a_2^2 + b_2^2}i$$

# The Complex Conjugate

▶ Start

$$\bullet \quad \overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$$

Let  $z = a_1 + b_1i$ ,  $w = a_2 + b_2i$  ( $a_1, a_2, b_1, b_2 \in \mathbb{R}$ )

$$\begin{aligned}\overline{\left(\frac{z}{w}\right)} &= \frac{\overline{a_1 + b_1i}}{a_2 + b_2i} = \frac{a_1a_2 - b_1b_2}{a_2^2 + b_2^2} + \frac{-a_1b_2 + a_2b_1}{a_2^2 + b_2^2}i \\ &= \frac{a_1a_2 - b_1b_2}{a_2^2 + b_2^2} - \frac{-a_1b_2 + a_2b_1}{a_2^2 + b_2^2}i\end{aligned}$$

# The Complex Conjugate

▶ Start

$$\bullet \overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$$

Let  $z = a_1 + b_1i$ ,  $w = a_2 + b_2i$  ( $a_1, a_2, b_1, b_2 \in \mathbb{R}$ )

$$\begin{aligned}\overline{\left(\frac{z}{w}\right)} &= \frac{\overline{a_1 + b_1i}}{a_2 + b_2i} = \frac{a_1a_2 - b_1b_2}{a_2^2 + b_2^2} + \frac{-a_1b_2 + a_2b_1}{a_2^2 + b_2^2}i \\ &= \frac{a_1a_2 - b_1b_2}{a_2^2 + b_2^2} - \frac{-a_1b_2 + a_2b_1}{a_2^2 + b_2^2}i \\ &= \frac{a_1a_2 - b_1b_2}{a_2^2 + b_2^2} + \frac{a_1b_2 - a_2b_1}{a_2^2 + b_2^2}i\end{aligned}$$

# The Complex Conjugate

▶ Start

$$\bullet \overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$$

Let  $z = a_1 + b_1i$ ,  $w = a_2 + b_2i$  ( $a_1, a_2, b_1, b_2 \in \mathbb{R}$ )

$$\begin{aligned}\overline{\left(\frac{z}{w}\right)} &= \frac{\overline{a_1 + b_1i}}{a_2 + b_2i} = \frac{a_1a_2 - b_1b_2}{a_2^2 + b_2^2} + \frac{-a_1b_2 + a_2b_1}{a_2^2 + b_2^2}i \\ &= \frac{a_1a_2 - b_1b_2}{a_2^2 + b_2^2} - \frac{-a_1b_2 + a_2b_1}{a_2^2 + b_2^2}i \\ &= \frac{a_1a_2 - b_1b_2}{a_2^2 + b_2^2} + \frac{a_1b_2 - a_2b_1}{a_2^2 + b_2^2}i \\ &= \frac{a_1 - b_1i}{a_2 - b_2i}\end{aligned}$$

# The Complex Conjugate

▶ Start

$$\bullet \quad \overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$$

Let  $z = a_1 + b_1i$ ,  $w = a_2 + b_2i$  ( $a_1, a_2, b_1, b_2 \in \mathbb{R}$ )

$$\begin{aligned}\overline{\left(\frac{z}{w}\right)} &= \frac{\overline{a_1 + b_1i}}{a_2 + b_2i} = \frac{a_1a_2 - b_1b_2}{a_2^2 + b_2^2} + \frac{-a_1b_2 + a_2b_1}{a_2^2 + b_2^2}i \\ &= \frac{a_1a_2 - b_1b_2}{a_2^2 + b_2^2} - \frac{-a_1b_2 + a_2b_1}{a_2^2 + b_2^2}i \\ &= \frac{a_1a_2 - b_1b_2}{a_2^2 + b_2^2} + \frac{a_1b_2 - a_2b_1}{a_2^2 + b_2^2}i \\ &= \frac{a_1 - b_1i}{a_2 - b_2i} = \frac{\overline{a_1 + b_1i}}{\overline{a_2 + b_2i}}\end{aligned}$$

# The Complex Conjugate

▶ Start

$$\bullet \quad \overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$$

Let  $z = a_1 + b_1i$ ,  $w = a_2 + b_2i$  ( $a_1, a_2, b_1, b_2 \in \mathbb{R}$ )

$$\begin{aligned}\overline{\left(\frac{z}{w}\right)} &= \frac{\overline{a_1 + b_1i}}{a_2 + b_2i} = \frac{a_1a_2 - b_1b_2}{a_2^2 + b_2^2} + \frac{-a_1b_2 + a_2b_1}{a_2^2 + b_2^2}i \\ &= \frac{a_1a_2 - b_1b_2}{a_2^2 + b_2^2} - \frac{-a_1b_2 + a_2b_1}{a_2^2 + b_2^2}i \\ &= \frac{a_1a_2 - b_1b_2}{a_2^2 + b_2^2} + \frac{a_1b_2 - a_2b_1}{a_2^2 + b_2^2}i \\ &= \frac{a_1 - b_1i}{a_2 - b_2i} = \frac{\overline{a_1 + b_1i}}{\overline{a_2 + b_2i}} = \frac{\bar{z}}{\bar{w}}\end{aligned}$$



# The Complex Conjugate

▶ Start

- $\bar{z} = z \Rightarrow z$  is a real number.

# The Complex Conjugate

▶ Start

- $\bar{z} = z \Rightarrow z$  is a real number.

Let  $z = a + bi (a, b \in \mathbb{R})$

# The Complex Conjugate

▶ Start

- $\bar{z} = z \Rightarrow z$  is a real number.

Let  $z = a + bi (a, b \in \mathbb{R})$

$$\bar{z} = z$$

# The Complex Conjugate

▶ Start

- $\bar{z} = z \Rightarrow z$  is a real number.

Let  $z = a + bi$  ( $a, b \in \mathbb{R}$ )

$$\begin{aligned}\bar{z} &= z \\ \overline{a + bi} &= a + bi\end{aligned}$$

# The Complex Conjugate

▶ Start

- $\bar{z} = z \Rightarrow z$  is a real number.

Let  $z = a + bi (a, b \in \mathbb{R})$

$$\begin{aligned}\bar{\bar{z}} &= z \\ \overline{a + bi} &= a - bi \\ a - bi &= a + bi\end{aligned}$$

# The Complex Conjugate

▶ Start

- $\bar{z} = z \Rightarrow z$  is a real number.

Let  $z = a + bi (a, b \in \mathbb{R})$

$$\begin{aligned}\bar{z} &= z \\ \overline{a + bi} &= a + bi \\ a - bi &= a + bi \\ 2bi &= 0\end{aligned}$$

# The Complex Conjugate

▶ Start

- $\bar{z} = z \Rightarrow z$  is a real number.

Let  $z = a + bi (a, b \in \mathbb{R})$

$$\begin{aligned}\bar{z} &= z \\ \overline{a + bi} &= a + bi \\ a - bi &= a + bi \\ 2bi &= 0 \\ b &= 0\end{aligned}$$

# The Complex Conjugate

▶ Start

- $\bar{z} = -z \Rightarrow z$  is an imaginary number.



# The Complex Conjugate

▶ Start

- $\bar{z} = -z \Rightarrow z$  is an imaginary number.

Let  $z = a + bi (a, b \in \mathbb{R})$

# The Complex Conjugate

▶ Start

- $\bar{z} = -z \Rightarrow z$  is an imaginary number.

Let  $z = a + bi (a, b \in \mathbb{R})$

$$\bar{z} = -z$$

# The Complex Conjugate

▶ Start

- $\bar{z} = -z \Rightarrow z$  is an imaginary number.

Let  $z = a + bi$  ( $a, b \in \mathbb{R}$ )

$$\begin{aligned}\bar{z} &= -z \\ \overline{a + bi} &= -(a + bi)\end{aligned}$$

# The Complex Conjugate

▶ Start

- $\bar{z} = -z \Rightarrow z$  is an imaginary number.

Let  $z = a + bi$  ( $a, b \in \mathbb{R}$ )

$$\begin{aligned}\bar{z} &= -z \\ \overline{a + bi} &= -(a + bi) \\ a - bi &= -a - bi\end{aligned}$$

# The Complex Conjugate

▶ Start

- $\bar{z} = -z \Rightarrow z$  is an imaginary number.

Let  $z = a + bi (a, b \in \mathbb{R})$

$$\begin{aligned}\bar{z} &= -z \\ \overline{a + bi} &= -(a + bi) \\ a - bi &= -a - bi \\ 2a &= 0\end{aligned}$$

# The Complex Conjugate

▶ Start

- $\bar{z} = -z \Rightarrow z$  is an imaginary number.

Let  $z = a + bi (a, b \in \mathbb{R})$

$$\begin{aligned}\bar{z} &= -z \\ \overline{a + bi} &= -(a + bi) \\ a - bi &= -a - bi \\ 2a &= 0 \\ a &= 0\end{aligned}$$