

# The Absolute Value of Real Numbers

▶ Start

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$$a \in \mathbb{R},$$

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$$a \in \mathbb{R}, |a|$$

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$$a \in \mathbb{R}, |a| = \left\{ \right.$$

# The Absolute Value of Real Numbers

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$$a \in \mathbb{R}, \quad |a| = \begin{cases} a \end{cases}$$

# The Absolute Value of Real Numbers

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$$a \in \mathbb{R}, \quad |a| = \begin{cases} a & , \quad \text{if } a \geq 0 \end{cases}$$

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$$a \in \mathbb{R}, \quad |a| = \begin{cases} a & , \quad \text{if } a \geq 0 \\ -a & \end{cases}$$

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$$a \in \mathbb{R}, \quad |a| = \begin{cases} a & , \quad \text{if } a \geq 0 \\ -a & , \quad \text{if } a < 0 \end{cases}$$



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$$a \in \mathbb{R}, \quad |a| = \begin{cases} a & , \quad \text{if } a \geq 0 \\ -a & , \quad \text{if } a < 0 \end{cases}$$

- $|a| = |-a|$

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- $|a| = |-a|$
- $|a|^2 = a^2, |a| = \sqrt{a^2}$

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- $|a| = |-a|$
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- $|ab| = |a||b|$
- $\left| \frac{a}{b} \right| = \frac{|a|}{|b|} \quad (b \neq 0)$

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$$a \in \mathbb{R}, |a| = \begin{cases} a & , \text{ if } a \geq 0 \\ -a & , \text{ if } a < 0 \end{cases}$$

- $|a| = |-a|$  ▶ Proof
- $|a|^2 = a^2, |a| = \sqrt{a^2}$  ▶ Proof
- $|ab| = |a||b|$  ▶ Proof
- $\left| \frac{a}{b} \right| = \frac{|a|}{|b|} \quad (b \neq 0)$  ▶ Proof

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$$|a| = |-a|$$

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$$|a| = |-a|$$

i)  $a > 0$

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i)  $a > 0$

$$|a|$$



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$$|a|^2 = (|a|)^2$$

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$$|a|^2 = |0|^2 = 0 = 0^2 = a^2$$

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$$|a|^2 = (|a|)^2 = (-a)^2$$

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**i)  $a > 0$**

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$$|ab| = |a||b|$$

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$$(|ab|)^2$$

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# The Absolute Value of Real Numbers

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$$|ab| = |a||b|$$

$$(|ab|)^2 = (ab)^2 = a^2b^2$$

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$$\left| \frac{a}{b} \right| = \frac{|a|}{|b|} \quad (b \neq 0)$$

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$$\left( \left| \frac{a}{b} \right| \right)^2$$

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$$\left( \left| \frac{a}{b} \right| \right)^2 = \left( \frac{a}{b} \right)^2$$

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$$\left| \frac{a}{b} \right| = \frac{|a|}{|b|} \quad (b \neq 0)$$

$$\left( \left| \frac{a}{b} \right| \right)^2 = \left( \frac{a}{b} \right)^2 = \frac{a^2}{b^2}$$



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$$\left| \frac{a}{b} \right| = \frac{|a|}{|b|} \quad (b \neq 0)$$

$$\left( \left| \frac{a}{b} \right| \right)^2 = \left( \frac{a}{b} \right)^2 = \frac{a^2}{b^2} = \frac{|a|^2}{|b|^2}$$

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$$\left( \left| \frac{a}{b} \right| \right)^2 = \left( \frac{a}{b} \right)^2 = \frac{a^2}{b^2} = \frac{|a|^2}{|b|^2} = \left( \frac{|a|}{|b|} \right)^2$$