Systems of Two Linear Equations in Two Variables

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One solution :

$$a_1b_2 \neq a_2b_1$$

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$$a_1b_2 \neq a_2b_1$$ Proof

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$$a_1b_2 - a_2b_1 \neq 0, a_1b_2 \neq a_2b_1$$

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$$a_1b_2 - a_2b_1 \neq 0, a_1b_2 \neq a_2b_1$$

$$x = -\frac{c_1b_2 - c_2b_1}{a_1b_2 - a_2b_1}, y = -\frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}$$

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No solutions : $a_1b_2 = a_2b_1$ and $a_1c_2 \neq a_2c_1$

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Infinitely many solutions : $a_1b_2 = a_2b_1$ and $a_1c_2 = a_2c_1$

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Start

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if $a_1 \neq 0$

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