

Solving Quadratic Inequalities in Algebra

$$ax^2 + bx + c \geq 0 \quad (a > 0, b, c \in \mathbb{R})$$

▶ Start

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$$\text{Let } D = b^2 - 4ac$$

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Let α and β

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$$x^2 + \frac{b}{a}x + \frac{c}{a} \geq 0 \quad (\because a > 0)$$

$$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} \geq 0$$

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$$\therefore \mathbb{R} \quad (\because b^2 - 4ac \leq 0)$$