

Polynomial Remainder Theorem

▶ Start

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- $f(x) : a \text{ polynomial}$

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- $f(x)$: a polynomial, $x - \alpha$

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- $f(x) : a \text{ polynomial}, x - \alpha$
 $\Rightarrow \exists Q(x) : a \text{ polynomial s.t. } f(x) = (x - \alpha)Q(x) + f(\alpha)$

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• $f(x)$: a polynomial, $x - \alpha$

$$\Rightarrow \exists Q(x) : \text{a polynomial s.t. } f(x) = (x - \alpha)Q(x) + f(\alpha)$$

$$\exists Q(x) : \text{a polynomial, } \exists R : \text{a number s.t. } f(x) = (x - \alpha)Q(x) + R$$

Polynomial Remainder Theorem

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• $f(x)$: a polynomial, $x - \alpha$

$$\Rightarrow \exists Q(x) : \text{a polynomial s.t. } f(x) = (x - \alpha)Q(x) + f(\alpha)$$

$$\begin{aligned} \exists Q(x) : \text{a polynomial, } \exists R : \text{a number s.t. } f(x) &= (x - \alpha)Q(x) + R \\ f(\alpha) &= (\alpha - \alpha)Q(\alpha) + R \end{aligned}$$

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$$f(\alpha) = (\alpha - \alpha)Q(\alpha) + R$$

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- $f(x) : a \text{ polynomial}$

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• $f(x)$: a polynomial, $ax + b$ ($a \neq 0$)

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- $f(x) : a \text{ polynomial}, ax + b \ (a \neq 0)$

$$\Rightarrow \exists Q(x) : a \text{ polynomial s.t. } f(x) = (ax + b)Q(x) + f\left(-\frac{b}{a}\right)$$

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$$f\left(-\frac{b}{a}\right) = \left\{ a \left(-\frac{b}{a} \right) + b \right\} Q\left(-\frac{b}{a}\right) + R$$

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