

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

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Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

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$$x_1 > 0, \dots, x_n > 0$$

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$$x_1 > 0, \dots, x_n > 0$$

$$\frac{x_1 + \dots + x_n}{n}$$

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$$x_1 > 0, \dots, x_n > 0$$

$$\frac{x_1 + \dots + x_n}{n} \geq$$

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$$x_1 > 0, \dots, x_n > 0$$

$$\frac{x_1 + \dots + x_n}{n} \geq \sqrt[n]{x_1 \times \dots \times x_n}$$

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$$x_1 > 0, \dots, x_n > 0$$

$$\frac{x_1 + \dots + x_n}{n} \geq \sqrt[n]{x_1 \times \dots \times x_n} \geq$$

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$$x_1 > 0, \dots, x_n > 0$$

$$\frac{x_1 + \dots + x_n}{n} \geq \sqrt[n]{x_1 \times \dots \times x_n} \geq \frac{n}{\frac{1}{x_1} + \dots + \frac{1}{x_n}}$$

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$$x_1 > 0, \dots, x_n > 0$$

$$\frac{x_1 + \dots + x_n}{n} \geq \sqrt[n]{x_1 \times \dots \times x_n} \geq \frac{n}{\frac{1}{x_1} + \dots + \frac{1}{x_n}}$$

▶ Proof of first inequality

▶ Proof of second inequality

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$$x_1 > 0, \dots, x_n > 0$$

$$\frac{x_1 + \dots + x_n}{n} \geq \sqrt[n]{x_1 \times \dots \times x_n} \geq \frac{n}{\frac{1}{x_1} + \dots + \frac{1}{x_n}}$$

▶ Proof of first inequality

▶ Proof of second inequality

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- Proof of first inequality

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- Proof of first inequality
If $n = 2$.

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- Proof of first inequality

If $n = 2$.

$$\left(\frac{x_1 + x_2}{2}\right)^2 - (\sqrt{x_1 x_2})^2$$

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- Proof of first inequality

If $n = 2$.

$$\left(\frac{x_1 + x_2}{2}\right)^2 - (\sqrt{x_1 x_2})^2 = \frac{x_1^2 + 2x_1 x_2 + x_2^2}{4} - x_1 x_2$$

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- Proof of first inequality

If $n = 2$.

$$\begin{aligned}\left(\frac{x_1 + x_2}{2}\right)^2 - (\sqrt{x_1 x_2})^2 &= \frac{x_1^2 + 2x_1 x_2 + x_2^2}{4} - x_1 x_2 \\ &= \frac{x_1^2 - 2x_1 x_2 + x_2^2}{4}\end{aligned}$$

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- Proof of first inequality

If $n = 2$.

$$\begin{aligned}\left(\frac{x_1 + x_2}{2}\right)^2 - (\sqrt{x_1 x_2})^2 &= \frac{x_1^2 + 2x_1 x_2 + x_2^2}{4} - x_1 x_2 \\ &= \frac{x_1^2 - 2x_1 x_2 + x_2^2}{4} \\ &= \frac{(x_1 - x_2)^2}{4}\end{aligned}$$

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- Proof of first inequality

If $n = 2$.

$$\begin{aligned}\left(\frac{x_1 + x_2}{2}\right)^2 - (\sqrt{x_1 x_2})^2 &= \frac{x_1^2 + 2x_1 x_2 + x_2^2}{4} - x_1 x_2 \\ &= \frac{x_1^2 - 2x_1 x_2 + x_2^2}{4} \\ &= \frac{(x_1 - x_2)^2}{4} \geq\end{aligned}$$

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- Proof of first inequality

If $n = 2$.

$$\begin{aligned}\left(\frac{x_1 + x_2}{2}\right)^2 - (\sqrt{x_1 x_2})^2 &= \frac{x_1^2 + 2x_1 x_2 + x_2^2}{4} - x_1 x_2 \\ &= \frac{x_1^2 - 2x_1 x_2 + x_2^2}{4} \\ &= \frac{(x_1 - x_2)^2}{4} \geq 0\end{aligned}$$

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- Proof of first inequality

If $n = 2$.

$$\left(\frac{x_1 + x_2}{2}\right)^2 - (\sqrt{x_1 x_2})^2 = \frac{x_1^2 + 2x_1 x_2 + x_2^2}{4} - x_1 x_2$$

$$= \frac{x_1^2 - 2x_1 x_2 + x_2^2}{4}$$

$$= \frac{(x_1 - x_2)^2}{4} \geq 0$$

$$\therefore \frac{x_1 + x_2}{2}$$

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- Proof of first inequality

If $n = 2$.

$$\left(\frac{x_1 + x_2}{2}\right)^2 - (\sqrt{x_1 x_2})^2 = \frac{x_1^2 + 2x_1 x_2 + x_2^2}{4} - x_1 x_2$$

$$= \frac{x_1^2 - 2x_1 x_2 + x_2^2}{4}$$

$$= \frac{(x_1 - x_2)^2}{4} \geq 0$$

$$\therefore \frac{x_1 + x_2}{2} \geq$$

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- Proof of first inequality

If $n = 2$.

$$\begin{aligned}\left(\frac{x_1 + x_2}{2}\right)^2 - (\sqrt{x_1 x_2})^2 &= \frac{x_1^2 + 2x_1 x_2 + x_2^2}{4} - x_1 x_2 \\ &= \frac{x_1^2 - 2x_1 x_2 + x_2^2}{4} \\ &= \frac{(x_1 - x_2)^2}{4} \geq 0\end{aligned}$$

$$\therefore \frac{x_1 + x_2}{2} \geq \sqrt{x_1 x_2}$$

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• Proof of first inequality

If $n = 2$.

$$\left(\frac{x_1 + x_2}{2}\right)^2 - (\sqrt{x_1 x_2})^2 = \frac{x_1^2 + 2x_1 x_2 + x_2^2}{4} - x_1 x_2$$

$$= \frac{x_1^2 - 2x_1 x_2 + x_2^2}{4}$$

$$= \frac{(x_1 - x_2)^2}{4} \geq 0$$

$$\therefore \frac{x_1 + x_2}{2} \geq \sqrt{x_1 x_2}$$

$$n = 2$$

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- Proof of first inequality

If $n = 2$.

$$\begin{aligned}\left(\frac{x_1 + x_2}{2}\right)^2 - (\sqrt{x_1 x_2})^2 &= \frac{x_1^2 + 2x_1 x_2 + x_2^2}{4} - x_1 x_2 \\ &= \frac{x_1^2 - 2x_1 x_2 + x_2^2}{4} \\ &= \frac{(x_1 - x_2)^2}{4} \geq 0\end{aligned}$$

$$\therefore \frac{x_1 + x_2}{2} \geq \sqrt{x_1 x_2}$$

$n = 2$ is true.

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- Proof of first inequality

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- Proof of first inequality

Assume

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- Proof of first inequality

Assume $n = 2^{k-1}$

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- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

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- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$\frac{x_1 + \cdots + x_{2^k}}{2^k}$$

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- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$\frac{x_1 + \cdots + x_{2^k}}{2^k} =$$

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- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$\frac{x_1 + \cdots + x_{2^k}}{2^k} = \frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}}$$

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- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$\frac{x_1 + \cdots + x_{2^k}}{2^k} = \frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} +$$

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- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$\frac{x_1 + \cdots + x_{2^k}}{2^k} = \frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}$$

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- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$\frac{x_1 + \cdots + x_{2^k}}{2^k} = \frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}$$

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- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$\frac{x_1 + \cdots + x_{2^k}}{2^k} = \frac{\frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2}$$
$$\geq$$

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- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$\begin{aligned}\frac{x_1 + \cdots + x_{2^k}}{2^k} &= \frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}} \\ &\geq \sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}}\end{aligned}$$

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- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$\begin{aligned}\frac{x_1 + \cdots + x_{2^k}}{2^k} &= \frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}} \\ &\geq 2^{k-1} \sqrt{x_1 \cdots x_{2^{k-1}}} +\end{aligned}$$

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- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$\begin{aligned} \frac{x_1 + \cdots + x_{2^k}}{2^k} &= \frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}} \\ &\geq 2^{k-1} \sqrt{x_1 \cdots x_{2^{k-1}}} + 2^{k-1} \sqrt{x_{2^{k-1}+1} \cdots x_{2^k}} \end{aligned}$$

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- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$\begin{aligned}\frac{x_1 + \cdots + x_{2^k}}{2^k} &= \frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}} \\ &\geq \frac{{}^{2^k-1}\sqrt{x_1 \cdots x_{2^{k-1}}} + {}^{2^k-1}\sqrt{x_{2^{k-1}+1} \cdots x_{2^k}}}{2}\end{aligned}$$

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- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$\begin{aligned}\frac{x_1 + \cdots + x_{2^k}}{2^k} &= \frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}} \\ &\geq \frac{{}^{2^{k-1}}\sqrt{x_1 \cdots x_{2^{k-1}}} + {}^{2^{k-1}}\sqrt{x_{2^{k-1}+1} \cdots x_{2^k}}}{2} \\ &\geq\end{aligned}$$

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- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$\begin{aligned}\frac{x_1 + \cdots + x_{2^k}}{2^k} &= \frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}} \\ &\geq \frac{2^{k-1}\sqrt{x_1 \cdots x_{2^{k-1}}} + 2^{k-1}\sqrt{x_{2^{k-1}+1} \cdots x_{2^k}}}{2} \\ &\geq \sqrt{\phantom{x_1 \cdots x_{2^k}}}\end{aligned}$$

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- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$\begin{aligned}\frac{x_1 + \cdots + x_{2^k}}{2^k} &= \frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}} \\ &\geq \frac{2^{k-1}\sqrt{x_1 \cdots x_{2^{k-1}}} + 2^{k-1}\sqrt{x_{2^{k-1}+1} \cdots x_{2^k}}}{2} \\ &\geq \sqrt{2^{k-1}\sqrt{x_1 \cdots x_{2^{k-1}}}}\end{aligned}$$

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- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$\begin{aligned}\frac{x_1 + \cdots + x_{2^k}}{2^k} &= \frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}} \\ &\geq \frac{2^{k-1}\sqrt{x_1 \cdots x_{2^{k-1}}} + 2^{k-1}\sqrt{x_{2^{k-1}+1} \cdots x_{2^k}}}{2} \\ &\geq \sqrt{2^{k-1}\sqrt{x_1 \cdots x_{2^{k-1}}} \cdot 2^{k-1}\sqrt{x_{2^{k-1}+1} \cdots x_{2^k}}}\end{aligned}$$

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- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$\begin{aligned}\frac{x_1 + \cdots + x_{2^k}}{2^k} &= \frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}} \\ &\geq \frac{{}^{2^{k-1}}\sqrt{x_1 \cdots x_{2^{k-1}}} + {}^{2^{k-1}}\sqrt{x_{2^{k-1}+1} \cdots x_{2^k}}}{2} \\ &\geq \sqrt{{}^{2^{k-1}}\sqrt{x_1 \cdots x_{2^{k-1}}} \cdot {}^{2^{k-1}}\sqrt{x_{2^{k-1}+1} \cdots x_{2^k}}} \\ &= \end{aligned}$$

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- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$\begin{aligned}\frac{x_1 + \cdots + x_{2^k}}{2^k} &= \frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}} \\ &\geq \frac{{}^{2^{k-1}}\sqrt{x_1 \cdots x_{2^{k-1}}} + {}^{2^{k-1}}\sqrt{x_{2^{k-1}+1} \cdots x_{2^k}}}{2} \\ &\geq \sqrt{{}^{2^k}\sqrt{x_1 \cdots x_{2^{k-1}}} \cdot {}^{2^k}\sqrt{x_{2^{k-1}+1} \cdots x_{2^k}}} \\ &= \sqrt[2^k]{x_1 \cdots x_{2^k}}\end{aligned}$$

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- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$\begin{aligned}\frac{x_1 + \cdots + x_{2^k}}{2^k} &= \frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}} \\ &\geq \frac{{}^{2^{k-1}}\sqrt{x_1 \cdots x_{2^{k-1}}} + {}^{2^{k-1}}\sqrt{x_{2^{k-1}+1} \cdots x_{2^k}}}{2} \\ &\geq \sqrt{{}^{2^{k-1}}\sqrt{x_1 \cdots x_{2^{k-1}}} \cdot {}^{2^{k-1}}\sqrt{x_{2^{k-1}+1} \cdots x_{2^k}}} \\ &= \sqrt[2^k]{x_1 \cdots x_{2^k}} \\ \therefore \frac{x_1 + \cdots + x_{2^k}}{2^k}\end{aligned}$$

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- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$\frac{x_1 + \cdots + x_{2^k}}{2^k} = \frac{\frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2}$$

$$\geq \frac{2^{k-1}\sqrt{x_1 \cdots x_{2^{k-1}}} + 2^{k-1}\sqrt{x_{2^{k-1}+1} \cdots x_{2^k}}}{2}$$

$$\geq \sqrt{2^{k-1}\sqrt{x_1 \cdots x_{2^{k-1}}} \cdot 2^{k-1}\sqrt{x_{2^{k-1}+1} \cdots x_{2^k}}}$$

$$= \sqrt[2^k]{x_1 \cdots x_{2^k}}$$

$$\therefore \frac{x_1 + \cdots + x_{2^k}}{2^k} \geq$$

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• Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$\frac{x_1 + \cdots + x_{2^k}}{2^k} = \frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}$$

$$\geq \frac{2^{k-1}\sqrt{x_1 \cdots x_{2^{k-1}}} + 2^{k-1}\sqrt{x_{2^{k-1}+1} \cdots x_{2^k}}}{2}$$

$$\geq \sqrt{2^{k-1}\sqrt{x_1 \cdots x_{2^{k-1}}} \cdot 2^{k-1}\sqrt{x_{2^{k-1}+1} \cdots x_{2^k}}}$$

$$= \sqrt[2^k]{x_1 \cdots x_{2^k}}$$

$$\therefore \frac{x_1 + \cdots + x_{2^k}}{2^k} \geq \sqrt[2^k]{x_1 \cdots x_{2^k}}$$

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• Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$\frac{x_1 + \cdots + x_{2^k}}{2^k} = \frac{\frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2}$$

$$\geq \frac{{}^{2^{k-1}}\sqrt{x_1 \cdots x_{2^{k-1}}} + {}^{2^{k-1}}\sqrt{x_{2^{k-1}+1} \cdots x_{2^k}}}{2}$$

$$\geq \sqrt{{}^{2^{k-1}}\sqrt{x_1 \cdots x_{2^{k-1}}} \cdot {}^{2^{k-1}}\sqrt{x_{2^{k-1}+1} \cdots x_{2^k}}}$$

$$= \sqrt[2^k]{x_1 \cdots x_{2^k}}$$

$$\therefore \frac{x_1 + \cdots + x_{2^k}}{2^k} \geq \sqrt[2^k]{x_1 \cdots x_{2^k}}$$

$$n = 2^k$$

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- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$\frac{x_1 + \cdots + x_{2^k}}{2^k} = \frac{\frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2}$$

$$\geq \frac{2^{k-1}\sqrt{x_1 \cdots x_{2^{k-1}}} + 2^{k-1}\sqrt{x_{2^{k-1}+1} \cdots x_{2^k}}}{2}$$

$$\geq \sqrt{2^{k-1}\sqrt{x_1 \cdots x_{2^{k-1}}} \cdot 2^{k-1}\sqrt{x_{2^{k-1}+1} \cdots x_{2^k}}}$$

$$= \sqrt[2^k]{x_1 \cdots x_{2^k}}$$

$$\therefore \frac{x_1 + \cdots + x_{2^k}}{2^k} \geq \sqrt[2^k]{x_1 \cdots x_{2^k}}$$

$n = 2^k$ is true.

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- Proof of first inequality

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

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- Proof of first inequality

Let $m = 2^l$

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- Proof of first inequality

Let $m = 2^l$ such that

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- Proof of first inequality

Let $m = 2^l$ such that $n < m$.

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- Proof of first inequality

Let $m = 2^l$ such that $n < m$.

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- Proof of first inequality

Let $m = 2^l$ such that $n < m$.

$$\alpha =$$

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- Proof of first inequality

Let $m = 2^l$ such that $n < m$.

$$\alpha = \frac{x_1 + \cdots + x_n}{n}$$

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- Proof of first inequality

Let $m = 2^l$ such that $n < m$.

$$\alpha = \frac{x_1 + \cdots + x_n}{n} =$$

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- Proof of first inequality

Let $m = 2^l$ such that $n < m$.

$$\alpha = \frac{x_1 + \cdots + x_n}{n} = \frac{m}{n}$$

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- Proof of first inequality

Let $m = 2^l$ such that $n < m$.

$$\alpha = \frac{x_1 + \cdots + x_n}{n} = \frac{m}{n}(x_1 + \cdots + x_n)$$

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- Proof of first inequality

Let $m = 2^l$ such that $n < m$.

$$\alpha = \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m}$$

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- Proof of first inequality

Let $m = 2^l$ such that $n < m$.

$$\alpha = \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m}$$

=

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- Proof of first inequality

Let $m = 2^l$ such that $n < m$.

$$\begin{aligned}\alpha &= \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m} \\ &= \frac{x_1 + \cdots + x_n}{m}\end{aligned}$$

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- Proof of first inequality

Let $m = 2^l$ such that $n < m$.

$$\begin{aligned}\alpha &= \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m} \\ &= \frac{x_1 + \cdots + x_n + \cdots + x_1}{m}\end{aligned}$$

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- Proof of first inequality

Let $m = 2^l$ such that $n < m$.

$$\begin{aligned}\alpha &= \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m} \\ &= x_1 + \cdots + x_n + \frac{m-n}{n}\end{aligned}$$

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

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- Proof of first inequality

Let $m = 2^l$ such that $n < m$.

$$\begin{aligned}\alpha &= \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{x_1 + \cdots + x_n + \frac{m-n}{n}(x_1 + \cdots + x_n)} \\ &= \end{aligned}$$

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Let $m = 2^l$ such that $n < m$.

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$$\alpha^m \geq x_1 \cdots x_n \alpha^{m-n},$$

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

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$$\alpha^m \geq x_1 \cdots x_n \alpha^{m-n}, \quad \alpha^n \geq x_1 \cdots x_n,$$

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Let $m = 2^l$ such that $n < m$.

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$$\alpha^m \geq x_1 \cdots x_n \alpha^{m-n}, \quad \alpha^n \geq x_1 \cdots x_n, \quad \alpha \geq \sqrt[n]{x_1 \cdots x_n}$$

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$$\alpha^m \geq x_1 \cdots x_n \alpha^{m-n}, \quad \alpha^n \geq x_1 \cdots x_n, \quad \alpha \geq \sqrt[n]{x_1 \cdots x_n}$$

$$\therefore \frac{x_1 + \cdots + x_n}{n} \geq \sqrt[n]{x_1 \cdots x_n} \quad (n \text{ is true.})$$

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- Proof of second inequality

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

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- Proof of second inequality

$$\frac{\frac{1}{x_1} + \dots + \frac{1}{x_n}}{n}$$

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$$\frac{\frac{1}{x_1} + \dots + \frac{1}{x_n}}{n} \geq$$

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- Proof of second inequality

$$\frac{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}{n} \geq \sqrt[n]{\frac{1}{x_1} \cdots \frac{1}{x_n}}$$

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$$\frac{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}{n} \geq \sqrt[n]{\frac{1}{x_1} \cdots \frac{1}{x_n}}$$

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$$\frac{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}{n} \geq \sqrt[n]{\frac{1}{x_1} \cdots \frac{1}{x_n}}$$

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$$\frac{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}{n} \geq \sqrt[n]{\frac{1}{x_1} \cdots \frac{1}{x_n}}$$

$$\frac{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}{n} \geq \frac{1}{\sqrt[n]{x_1 \cdots x_n}}$$

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

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$$\frac{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}{n} \geq \sqrt[n]{\frac{1}{x_1} \cdots \frac{1}{x_n}}$$

$$\frac{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}{n} \geq \frac{1}{\sqrt[n]{x_1 \cdots x_n}}$$

$$\therefore \sqrt[n]{x_1 \times \cdots \times x_n} \geq$$

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$$\frac{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}{n} \geq \sqrt[n]{\frac{1}{x_1} \cdots \frac{1}{x_n}}$$

$$\frac{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}{n} \geq \frac{1}{\sqrt[n]{x_1 \cdots x_n}}$$

$$\therefore \sqrt[n]{x_1 \times \cdots \times x_n} \geq \frac{n}{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}$$

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$$\frac{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}{n} \geq \sqrt[n]{\frac{1}{x_1} \cdots \frac{1}{x_n}}$$

$$\frac{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}{n} \geq \frac{1}{\sqrt[n]{x_1 \cdots x_n}}$$

$$\therefore \sqrt[n]{x_1 \times \cdots \times x_n} \geq \frac{n}{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}$$