

Function Composition

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Function composition

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Function composition is

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Function composition is the pointwise application

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Function Composition

▶ Start

Function Composition

▶ Start

X

Function Composition

▶ Start

$$\mathbf{X} \xrightarrow{f}$$

Function Composition

▶ Start

$$X \xrightarrow{f} Y$$

Function Composition

▶ Start

$$X \xrightarrow{f} Y \xrightarrow{g}$$

Function Composition

▶ Start

$$X \xrightarrow{f} Y \xrightarrow{g} Z$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} \\ x & & & & \end{array}$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} \\ x & \xrightarrow{f} & & & \end{array}$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} \\ x & \xrightarrow{f} & y & & \end{array}$$

Function Composition

▶ Start

$$\begin{array}{ccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & \end{array}$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \end{array}$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & & & & \end{array}$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & \xrightarrow{f} & & & \end{array}$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & \xrightarrow{f} & f(x) & & \end{array}$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & \end{array}$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) \end{array}$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & \end{array}$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & & \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & & \end{array}$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) \end{array}$$

$$\mathbf{X} \xrightarrow{g \circ f} \mathbf{Z}$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & & \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & & & \end{array}$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) \end{array}$$

$$\begin{array}{ccc} \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{g \circ f} & \end{array}$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) \end{array}$$

$$\begin{array}{ccc} \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{g \circ f} & z \end{array}$$

Function Composition

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$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) \end{array}$$

$$\begin{array}{ccc} \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{g \circ f} & z \\ x & & \end{array}$$

Function Composition

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Function Composition

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$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) \end{array}$$

$$\begin{array}{ccc} \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) \\ & & & & \therefore \end{array}$$

$$\begin{array}{ccc} \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) \\ & & \therefore \forall x & & \end{array}$$

$$\begin{array}{ccc} \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

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$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) \end{array}$$

$\therefore \forall x \in \mathbf{X},$

$$\begin{array}{ccc} \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

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$$\begin{array}{ccc} \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) \end{array}$$

$\therefore \forall x \in \mathbf{X}, (g$

$$\begin{array}{ccc} \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

Function Composition

▶ Start

$$\begin{array}{lclcl} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) \end{array}$$

$\therefore \forall x \in \mathbf{X}, (g \circ$

$$\begin{array}{lcl} \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) \end{array} \qquad \begin{array}{ccc} \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$\therefore \forall x \in \mathbf{X}, (g \circ f$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$\therefore \forall x \in \mathbf{X}, (g \circ f)$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$\therefore \forall x \in \mathbf{X}, (g \circ f)(x)$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) =$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
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Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

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Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

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Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

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Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$((h \circ$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$((h \circ g$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$((h \circ g)$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x)$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) =$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = ($$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)($$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x))$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x))$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x))$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(f(x))$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

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Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
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$$= h($$

Function Composition

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$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
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$$= h(($$

Function Composition

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$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
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Function Composition

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Function Composition

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$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
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Function Composition

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$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
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Function Composition

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Function Composition

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Function Composition

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$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
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Function Composition

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Function Composition

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$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
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Function Composition

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$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
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Function Composition

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$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
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\therefore

Function Composition

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$$\therefore \forall x$$

Function Composition

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Function Composition

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$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
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Function Composition

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Function Composition

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Function Composition

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Function Composition

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Function Composition

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Function Composition

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Function Composition

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Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$\begin{aligned} ((h \circ g) \circ f)(x) &= (h \circ g)(f(x)) = h(g(f(x))) \\ &= h((g \circ f)(x)) = (h \circ (g \circ f))(x) \end{aligned}$$

$$\therefore \forall x \in \mathbf{X}, ((h \circ g) \circ f)(x)$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore \forall x \in \mathbf{X}, ((h \circ g) \circ f)(x) =$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
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Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
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Function Composition

▶ Start

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Function Composition

▶ Start

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▶ Start

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▶ Start

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Function Composition

▶ Start

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Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
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Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
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Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
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Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

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$$\therefore \forall x \in \mathbf{X}, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

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Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore \forall x \in \mathbf{X}, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

$$(h \circ$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

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$$\therefore \forall x \in \mathbf{X}, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

$$(h \circ g)$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

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$$\therefore \forall x \in \mathbf{X}, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

$$(h \circ g)$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
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$$\therefore \forall x \in \mathbf{X}, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

$$(h \circ g) \circ$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
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$$\therefore \forall x \in \mathbf{X}, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

$$(h \circ g) \circ f$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
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$$\therefore \forall x \in \mathbf{X}, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

$$(h \circ g) \circ f =$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
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$$\therefore \forall x \in \mathbf{X}, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

$$(h \circ g) \circ f = h \circ (g \circ f)$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
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Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
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$$(h \circ g) \circ f = h \circ (g \circ f)$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
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$$(h \circ g) \circ f = h \circ (g \circ f)$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
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$$(h \circ g) \circ f = h \circ (g \circ f)$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
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$$(h \circ g) \circ f = h \circ (g \circ f)$$

Function Composition

▶ Start

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$
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$$(h \circ g) \circ f = h \circ (g \circ f)$$

Function Composition

▶ Home

$$\begin{array}{ccccc} \mathbf{X} & \xrightarrow{f} & \mathbf{Y} & \xrightarrow{g} & \mathbf{Z} & & \mathbf{X} & \xrightarrow{g \circ f} & \mathbf{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore \forall x \in \mathbf{X}, (g \circ f)(x) = g(f(x))$$

$$\begin{aligned} ((h \circ g) \circ f)(x) &= (h \circ g)(f(x)) = h(g(f(x))) \\ &= h((g \circ f)(x)) = (h \circ (g \circ f))(x) \end{aligned}$$

$$\therefore \forall x \in \mathbf{X}, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

$$(h \circ g) \circ f = h \circ (g \circ f)$$