

Discriminant of the Quadratic Formula

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$$ax^2 + bx + c = 0 \quad (a \neq 0)$$

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$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

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- $a, b, c \in \mathbb{R}$

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$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

- $a, b, c \in \mathbb{R}$
 - $b^2 - 4ac > 0$

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- $a, b, c \in \mathbb{R}$
 - $b^2 - 4ac > 0$: Two distinct real roots

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 - $b^2 - 4ac = 0$

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- $a, b, c \in \mathbb{R}$
 - $b^2 - 4ac > 0$: Two distinct real roots
 - $b^2 - 4ac = 0$: Two coincident real roots

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- $a, b, c \in \mathbb{R}$
 - $b^2 - 4ac > 0$: Two distinct real roots
 - $b^2 - 4ac = 0$: Two coincident real roots
 - $b^2 - 4ac < 0$

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 - $b^2 - 4ac = 0$: Two coincident real roots
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- $a, b, c \in \mathbb{C}$

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$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

- $a, b, c \in \mathbb{R}$
 - $b^2 - 4ac > 0$: Two distinct real roots
 - $b^2 - 4ac = 0$: Two coincident real roots
 - $b^2 - 4ac < 0$: Two distinct complex roots which are not real
- $a, b, c \in \mathbb{C}$
 - $b^2 - 4ac \neq 0$

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- $a, b, c \in \mathbb{C}$
 - $b^2 - 4ac \neq 0$: Two distinct roots
 - $b^2 - 4ac = 0$

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$$ax^2 + bx + c = 0 \quad (a \neq 0)$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2} \quad \text{▶ Proof}$$

- $a, b, c \in \mathbb{R}$
 - $b^2 - 4ac > 0$: Two distinct real roots
 - $b^2 - 4ac = 0$: Two coincident real roots
 - $b^2 - 4ac < 0$: Two distinct complex roots
- $a, b, c \in \mathbb{C}$
 - $b^2 - 4ac \neq 0$: Two distinct roots
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$$ax^2 + bx + c = 0$$

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$$ax^2 + bx + c = 0$$
$$a \left(x^2 + \frac{b}{a}x \right) + c = 0$$

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$$\begin{aligned}ax^2 + bx + c &= 0 \\a \left(x^2 + \frac{b}{a}x \right) + c &= 0 \\a \left(x^2 + 2\frac{b}{2a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} \right) + c &= 0\end{aligned}$$

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