

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

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▶ Start

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▶ Start

$$\sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(at + b)^{n-1}$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(at + b)^{n-1}$$
$$\sum_{r=1}^n$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\begin{aligned} \sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r &= nat(at + b)^{n-1} \\ \sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} & \end{aligned}$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(at + b)^{n-1}$$

$$\sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} = na$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(at + b)^{n-1}$$

$$\sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} = na(at + b)^{n-1}$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(at + b)^{n-1}$$

$$\sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} = na(at + b)^{n-1} + nat$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(at + b)^{n-1}$$

$$\sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} = na(at + b)^{n-1} + nat(n-1)(at + b)^{n-2}a$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(at + b)^{n-1}$$

$$\begin{aligned} \sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at + b)^{n-1} + nat(n-1)(at + b)^{n-2}a \\ &= na(at + b)^{n-2} \end{aligned}$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(at + b)^{n-1}$$

$$\begin{aligned} \sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at + b)^{n-1} + nat(n-1)(at + b)^{n-2}a \\ &= na(at + b)^{n-2} \{(at + b)\} \end{aligned}$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(at + b)^{n-1}$$

$$\begin{aligned} \sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at + b)^{n-1} + nat(n-1)(at + b)^{n-2}a \\ &= na(at + b)^{n-2} \{(at + b) + t(n-1)a\} \end{aligned}$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(at + b)^{n-1}$$

$$\begin{aligned} \sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at + b)^{n-1} + nat(n-1)(at + b)^{n-2}a \\ &= na(at + b)^{n-2} \{(at + b) + t(n-1)a\} \\ &= na(at + b)^{n-2} \end{aligned}$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(at + b)^{n-1}$$

$$\begin{aligned} \sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at + b)^{n-1} + nat(n-1)(at + b)^{n-2}a \\ &= na(at + b)^{n-2} \{(at + b) + t(n-1)a\} \\ &= na(at + b)^{n-2}(at + b + nat - at) \end{aligned}$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\begin{aligned} \sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r &= nat(at + b)^{n-1} \\ \sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at + b)^{n-1} + nat(n-1)(at + b)^{n-2}a \\ &= na(at + b)^{n-2} \{(at + b) + t(n-1)a\} \\ &= na(at + b)^{n-2}(at + b + nat - at) \\ &= na(at + b)^{n-2} \end{aligned}$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\begin{aligned} \sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r &= nat(at + b)^{n-1} \\ \sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at + b)^{n-1} + nat(n-1)(at + b)^{n-2}a \\ &= na(at + b)^{n-2} \{(at + b) + t(n-1)a\} \\ &= na(at + b)^{n-2}(at + b + nat - at) \\ &= na(at + b)^{n-2}(b + nat) \end{aligned}$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\begin{aligned} \sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r &= nat(at + b)^{n-1} \\ \sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at + b)^{n-1} + nat(n-1)(at + b)^{n-2}a \\ &= na(at + b)^{n-2} \{(at + b) + t(n-1)a\} \\ &= na(at + b)^{n-2}(at + b + nat - at) \\ &= na(at + b)^{n-2}(b + nat) = na(at + b)^{n-2} \end{aligned}$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\begin{aligned} \sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r &= nat(at + b)^{n-1} \\ \sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at + b)^{n-1} + nat(n-1)(at + b)^{n-2}a \\ &= na(at + b)^{n-2} \{(at + b) + t(n-1)a\} \\ &= na(at + b)^{n-2}(at + b + nat - at) \\ &= na(at + b)^{n-2}(b + nat) = na(at + b)^{n-2}(nat + b) \end{aligned}$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\begin{aligned} \sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r &= nat(at + b)^{n-1} \\ \sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at + b)^{n-1} + nat(n-1)(at + b)^{n-2}a \\ &= na(at + b)^{n-2} \{ (at + b) + t(n-1)a \} \\ &= na(at + b)^{n-2} (at + b + nat - at) \\ &= na(at + b)^{n-2} (b + nat) = na(at + b)^{n-2} (nat + b) \\ &= na(nat + b)(at + b)^{n-2} \end{aligned}$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(at + b)^{n-1}$$

$$\begin{aligned} \sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at + b)^{n-1} + nat(n-1)(at + b)^{n-2}a \\ &= na(at + b)^{n-2} \{ (at + b) + t(n-1)a \} \\ &= na(at + b)^{n-2} (at + b + nat - at) \\ &= na(at + b)^{n-2} (b + nat) = na(at + b)^{n-2} (nat + b) \\ &= na(nat + b)(at + b)^{n-2} \end{aligned}$$

$$\sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r}$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(at + b)^{n-1}$$

$$\begin{aligned} \sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at + b)^{n-1} + nat(n-1)(at + b)^{n-2}a \\ &= na(at + b)^{n-2} \{ (at + b) + t(n-1)a \} \\ &= na(at + b)^{n-2} (at + b + nat - at) \\ &= na(at + b)^{n-2} (b + nat) = na(at + b)^{n-2} (nat + b) \\ &= na(nat + b)(at + b)^{n-2} \end{aligned}$$

$$\sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(at + b)^{n-1}$$

$$\begin{aligned} \sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at + b)^{n-1} + nat(n-1)(at + b)^{n-2}a \\ &= na(at + b)^{n-2} \{ (at + b) + t(n-1)a \} \\ &= na(at + b)^{n-2} (at + b + nat - at) \\ &= na(at + b)^{n-2} (b + nat) = na(at + b)^{n-2} (nat + b) \\ &= na(nat + b)(at + b)^{n-2} \end{aligned}$$

$$\sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r = na$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(at + b)^{n-1}$$

$$\begin{aligned} \sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at + b)^{n-1} + nat(n-1)(at + b)^{n-2}a \\ &= na(at + b)^{n-2} \{ (at + b) + t(n-1)a \} \\ &= na(at + b)^{n-2} (at + b + nat - at) \\ &= na(at + b)^{n-2} (b + nat) = na(at + b)^{n-2} (nat + b) \\ &= na(nat + b)(at + b)^{n-2} \end{aligned}$$

$$\sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(at + b)^{n-1}$$

$$\begin{aligned} \sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at + b)^{n-1} + nat(n-1)(at + b)^{n-2}a \\ &= na(at + b)^{n-2} \{ (at + b) + t(n-1)a \} \\ &= na(at + b)^{n-2} (at + b + nat - at) \\ &= na(at + b)^{n-2} (b + nat) = na(at + b)^{n-2} (nat + b) \\ &= na(nat + b)(at + b)^{n-2} \end{aligned}$$

$$\sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(nat + b)(at + b)^{n-2}$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(at + b)^{n-1}$$

$$\begin{aligned} \sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at + b)^{n-1} + nat(n-1)(at + b)^{n-2}a \\ &= na(at + b)^{n-2} \{ (at + b) + t(n-1)a \} \\ &= na(at + b)^{n-2} (at + b + nat - at) \\ &= na(at + b)^{n-2} (b + nat) = na(at + b)^{n-2} (nat + b) \\ &= na(nat + b)(at + b)^{n-2} \end{aligned}$$

$$\sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(nat + b)(at + b)^{n-2} = \sum_{r=0}^n$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(at + b)^{n-1}$$

$$\begin{aligned} \sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at + b)^{n-1} + nat(n-1)(at + b)^{n-2}a \\ &= na(at + b)^{n-2} \{ (at + b) + t(n-1)a \} \\ &= na(at + b)^{n-2} (at + b + nat - at) \\ &= na(at + b)^{n-2} (b + nat) = na(at + b)^{n-2} (nat + b) \\ &= na(nat + b)(at + b)^{n-2} \end{aligned}$$

$$\sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(nat + b)(at + b)^{n-2} = \sum_{r=0}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(at + b)^{n-1}$$

$$\begin{aligned} \sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at + b)^{n-1} + nat(n-1)(at + b)^{n-2}a \\ &= na(at + b)^{n-2} \{ (at + b) + t(n-1)a \} \\ &= na(at + b)^{n-2} (at + b + nat - at) \\ &= na(at + b)^{n-2} (b + nat) = na(at + b)^{n-2} (nat + b) \\ &= na(nat + b)(at + b)^{n-2} \end{aligned}$$

$$\sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(nat + b)(at + b)^{n-2} = \sum_{r=0}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r$$

∴

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(at + b)^{n-1}$$

$$\begin{aligned} \sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at + b)^{n-1} + nat(n-1)(at + b)^{n-2}a \\ &= na(at + b)^{n-2} \{ (at + b) + t(n-1)a \} \\ &= na(at + b)^{n-2} (at + b + nat - at) \\ &= na(at + b)^{n-2} (b + nat) = na(at + b)^{n-2} (nat + b) \\ &= na(nat + b)(at + b)^{n-2} \end{aligned}$$

$$\sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(nat + b)(at + b)^{n-2} = \sum_{r=0}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r$$

$$\therefore \sum_{r=0}^n$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(at + b)^{n-1}$$

$$\begin{aligned} \sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at + b)^{n-1} + nat(n-1)(at + b)^{n-2}a \\ &= na(at + b)^{n-2} \{ (at + b) + t(n-1)a \} \\ &= na(at + b)^{n-2} (at + b + nat - at) \\ &= na(at + b)^{n-2} (b + nat) = na(at + b)^{n-2} (nat + b) \\ &= na(nat + b)(at + b)^{n-2} \end{aligned}$$

$$\sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(nat + b)(at + b)^{n-2} = \sum_{r=0}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r$$

$$\therefore \sum_{r=0}^n r^2$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(at + b)^{n-1}$$

$$\begin{aligned} \sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at + b)^{n-1} + nat(n-1)(at + b)^{n-2}a \\ &= na(at + b)^{n-2} \{ (at + b) + t(n-1)a \} \\ &= na(at + b)^{n-2} (at + b + nat - at) \\ &= na(at + b)^{n-2} (b + nat) = na(at + b)^{n-2} (nat + b) \\ &= na(nat + b)(at + b)^{n-2} \end{aligned}$$

$$\sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(nat + b)(at + b)^{n-2} = \sum_{r=0}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r$$

$$\therefore \sum_{r=0}^n r^2 \cdot {}_n C_r$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(at + b)^{n-1}$$

$$\begin{aligned} \sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at + b)^{n-1} + nat(n-1)(at + b)^{n-2}a \\ &= na(at + b)^{n-2} \{ (at + b) + t(n-1)a \} \\ &= na(at + b)^{n-2} (at + b + nat - at) \\ &= na(at + b)^{n-2} (b + nat) = na(at + b)^{n-2} (nat + b) \\ &= na(nat + b)(at + b)^{n-2} \end{aligned}$$

$$\sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(nat + b)(at + b)^{n-2} = \sum_{r=0}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r$$

$$\therefore \sum_{r=0}^n r^2 \cdot {}_n C_r a^r$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(at + b)^{n-1}$$

$$\begin{aligned} \sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at + b)^{n-1} + nat(n-1)(at + b)^{n-2}a \\ &= na(at + b)^{n-2} \{ (at + b) + t(n-1)a \} \\ &= na(at + b)^{n-2} (at + b + nat - at) \\ &= na(at + b)^{n-2} (b + nat) = na(at + b)^{n-2} (nat + b) \\ &= na(nat + b)(at + b)^{n-2} \end{aligned}$$

$$\sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(nat + b)(at + b)^{n-2} = \sum_{r=0}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r$$

$$\therefore \sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r}$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(at + b)^{n-1}$$

$$\begin{aligned} \sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at + b)^{n-1} + nat(n-1)(at + b)^{n-2}a \\ &= na(at + b)^{n-2} \{ (at + b) + t(n-1)a \} \\ &= na(at + b)^{n-2} (at + b + nat - at) \\ &= na(at + b)^{n-2} (b + nat) = na(at + b)^{n-2} (nat + b) \\ &= na(nat + b)(at + b)^{n-2} \end{aligned}$$

$$\sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(nat + b)(at + b)^{n-2} = \sum_{r=0}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r$$

$$\therefore \sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(at + b)^{n-1}$$

$$\begin{aligned} \sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at + b)^{n-1} + nat(n-1)(at + b)^{n-2}a \\ &= na(at + b)^{n-2} \{ (at + b) + t(n-1)a \} \\ &= na(at + b)^{n-2} (at + b + nat - at) \\ &= na(at + b)^{n-2} (b + nat) = na(at + b)^{n-2} (nat + b) \\ &= na(nat + b)(at + b)^{n-2} \end{aligned}$$

$$\sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(nat + b)(at + b)^{n-2} = \sum_{r=0}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r$$

$$\therefore \sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

▶ Start

$$\sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(at + b)^{n-1}$$

$$\begin{aligned} \sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at + b)^{n-1} + nat(n-1)(at + b)^{n-2}a \\ &= na(at + b)^{n-2} \{ (at + b) + t(n-1)a \} \\ &= na(at + b)^{n-2} (at + b + nat - at) \\ &= na(at + b)^{n-2} (b + nat) = na(at + b)^{n-2} (nat + b) \\ &= na(nat + b)(at + b)^{n-2} \end{aligned}$$

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▶ Start

$$\sum_{r=0}^n r \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(at + b)^{n-1}$$

$$\begin{aligned} \sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at + b)^{n-1} + nat(n-1)(at + b)^{n-2}a \\ &= na(at + b)^{n-2} \{ (at + b) + t(n-1)a \} \\ &= na(at + b)^{n-2} (at + b + nat - at) \\ &= na(at + b)^{n-2} (b + nat) = na(at + b)^{n-2} (nat + b) \\ &= na(nat + b)(at + b)^{n-2} \end{aligned}$$

$$\sum_{r=1}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(nat + b)(at + b)^{n-2} = \sum_{r=0}^n r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r$$

$$\therefore \sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

$$\sum_{r=0}^n r^2 \cdot {}_n C_r a^r b^{n-r} = na(na + b)(a + b)^{n-2}$$

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END