연쇄법칙
(The Chain Rule)

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Theorem

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$$F'(x)$$

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$$\begin{aligned} f(u+k) - f(u) &= f'(u) \cdot k + k \cdot \varepsilon_2(k) \\ &\quad (\text{Let } k = g(x+h) - g(x) \ , \ g(x+h) = g(x) + k = u + k) \\ f(g(x+h)) - f(g(x)) &= \{f'(u) + \varepsilon_2(g(x+h) - g(x))\}\{g(x+h) - g(x)\} \\ &= \{f'(u) + \varepsilon_2(g(x+h) - g(x))\}\{g'(x) + \varepsilon_1(h)\} \cdot h \end{aligned}$$

$$F'(x) = \lim_{h \to 0} \frac{f(g(x+h)) - f(g(x))}{h}$$

Start

Proof.

$$\varepsilon_1(h) = \begin{cases} \dfrac{g(x+h)-g(x)}{h} - g'(x) & , \quad h \neq 0 \\ 0 & , \quad h = 0 \end{cases}$$

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▸ Start

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