

## 압착정리 (The Squeeze Theorem)

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## Theorem

# The Squeeze Theorem

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$$f(x) \leq g(x) \leq h(x) (0 < |x - a| < \delta_0), \lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} h(x) = L$$

$$\lim_{x \rightarrow a} g(x) = L$$

## Proof.

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