나눗셈의 극한은 극한의 나눗셈이다.

(The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).)

Start

Theorem

Start

Theorem

$\lim_{x \longrightarrow a} f(x) = L$

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$\lim_{x \longrightarrow a} f(x) = L$, $\lim_{x \longrightarrow a} g(x) = M(\neq 0)$

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