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The limit of a quotient is the quotient of the limits(provided that the limit of the denominator is not 0).

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END