

## 함수 (Function)

▶ Start

▶ Start

## Definition (Function)

▶ Start

## Definition (Function)

A **function**  $f$

▶ Start

## Definition (Function)

A **function**  $f$  is

▶ Start

## Definition (Function)

A **function**  $f$  is a rule

▶ Start

## Definition (Function)

A **function**  $f$  is a rule that assigns to each element  $x$

▶ Start

## Definition (Function)

A **function**  $f$  is a rule that assigns to each element  $x$  in a set  $D$



▶ Start

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▶ Start

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▶ Start

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▶ Start

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▶ Start

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▶ Start

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A symbol

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A symbol that represents an arbitrary number

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▶ home

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▶ Start

▶ Start

A function  $f$

▶ Start

A function  $f$  from  $X$

▶ Start

A function  $f$  from  $X$  to  $Y$

▶ Start

A function  $f$  from  $X$  to  $Y$  is a subset

▶ Start

A function  $f$  from  $X$  to  $Y$  is a subset of the Cartesian product  $X \times Y$

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A function  $f$  from  $X$  to  $Y$  is a subset of the Cartesian product  $X \times Y$  subject



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Every element

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Every element of  $X$

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A function  $f$  from  $X$  to  $Y$  is a subset of the Cartesian product  $X \times Y$  subject to the following condition:  
Every element of  $X$  is the first component

▶ Start

A function  $f$  from  $X$  to  $Y$  is a subset of the Cartesian product  $X \times Y$  subject to the following condition:

Every element of  $X$  is the first component of one and only one

▶ Start

A function  $f$  from  $X$  to  $Y$  is a subset of the Cartesian product  $X \times Y$  subject to the following condition:

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▶ Start

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In other words,



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▶ Start

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▶ home

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▶ Start

▶ Start

$X \times Y$

▶ Start

$$\mathbf{X} \times \mathbf{Y} =$$

▶ Start

$$\mathbf{X} \times \mathbf{Y} = \{$$

▶ Start

$$\mathbf{X} \times \mathbf{Y} = \{(x, y)\}$$



▶ Start

$$\mathbf{X} \times \mathbf{Y} = \{(x, y) |$$

▶ Start

$$\mathbf{X} \times \mathbf{Y} = \{(x, y) | x \in \mathbf{X},$$

▶ Start

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▶ Start

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▶ Start

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▶ Start

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▶ Start

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$f$

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$$f : \mathbf{X}$$

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$$f : \mathbf{X} \rightarrow$$

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$$f : \mathbf{X} \rightarrow \mathbf{Y},$$

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$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X}$$

▶ Start

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$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f}$$



▶ Start

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$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

▶ Start

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$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

$\mathbf{X}$

▶ Start

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$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

**X** Domain

▶ Start

$\mathbf{X} \times \mathbf{Y} = \{(x, y) | x \in \mathbf{X}, y \in \mathbf{Y}\}$  : The Cartesian product  $\mathbf{X} \times \mathbf{Y}$   
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$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

$\mathbf{X}$  Domain

$\mathbf{Y}$

▶ Start

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$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

$\mathbf{X}$  Domain

$\mathbf{Y}$  Codomain

▶ Start

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$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

**X** Domain

**Y** Codomain

$f$

▶ Start

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$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

**X** Domain

**Y** Codomain

$f :$

▶ Start

$\mathbf{X} \times \mathbf{Y} = \{(x, y) | x \in \mathbf{X}, y \in \mathbf{Y}\}$  : The Cartesian product  $\mathbf{X} \times \mathbf{Y}$   
 $f$  is a function from  $\mathbf{X}$  to  $\mathbf{Y}$ .

$$\left\{ \begin{array}{l} f \subset \mathbf{X} \times \mathbf{Y} \\ \forall x \in \mathbf{X}, \exists! y \in \mathbf{Y} \text{ s.t. } (x, y) \in f \end{array} \right\} \left\{ \begin{array}{l} \forall x \in \mathbf{X}, \exists y \in \mathbf{Y} \text{ s.t. } (x, y) \in f \\ (x, y_1) \in f \text{ and } (x, y_2) \in f \Rightarrow y_1 = y_2 \end{array} \right.$$

$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

**X** Domain

**Y** Codomain

$$f : x$$



▶ Start

$\mathbf{X} \times \mathbf{Y} = \{(x, y) | x \in \mathbf{X}, y \in \mathbf{Y}\}$  : The Cartesian product  $\mathbf{X} \times \mathbf{Y}$   
 $f$  is a function from  $\mathbf{X}$  to  $\mathbf{Y}$ .

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$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

**X** Domain

**Y** Codomain

$$f : x \longrightarrow$$

▶ Start

$\mathbf{X} \times \mathbf{Y} = \{(x, y) | x \in \mathbf{X}, y \in \mathbf{Y}\}$  : The Cartesian product  $\mathbf{X} \times \mathbf{Y}$   
 $f$  is a function from  $\mathbf{X}$  to  $\mathbf{Y}$ .

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$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

**X** Domain

**Y** Codomain

$$f : x \longrightarrow y,$$

▶ Start

$\mathbf{X} \times \mathbf{Y} = \{(x, y) | x \in \mathbf{X}, y \in \mathbf{Y}\}$  : The Cartesian product  $\mathbf{X} \times \mathbf{Y}$   
 $f$  is a function from  $\mathbf{X}$  to  $\mathbf{Y}$ .

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$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

**X** Domain

**Y** Codomain

$$f : x \longrightarrow y, x$$

▶ Start

$\mathbf{X} \times \mathbf{Y} = \{(x, y) | x \in \mathbf{X}, y \in \mathbf{Y}\}$  : The Cartesian product  $\mathbf{X} \times \mathbf{Y}$   
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$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

**X** Domain

**Y** Codomain

$$f : x \longrightarrow y, x \xrightarrow{f}$$

▶ Start

$\mathbf{X} \times \mathbf{Y} = \{(x, y) | x \in \mathbf{X}, y \in \mathbf{Y}\}$  : The Cartesian product  $\mathbf{X} \times \mathbf{Y}$   
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$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

**X** Domain

**Y** Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y,$$

▶ Start

$\mathbf{X} \times \mathbf{Y} = \{(x, y) | x \in \mathbf{X}, y \in \mathbf{Y}\}$  : The Cartesian product  $\mathbf{X} \times \mathbf{Y}$   
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$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

**X** Domain

**Y** Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y$$

▶ Start

$\mathbf{X} \times \mathbf{Y} = \{(x, y) | x \in \mathbf{X}, y \in \mathbf{Y}\}$  : The Cartesian product  $\mathbf{X} \times \mathbf{Y}$   
 $f$  is a function from  $\mathbf{X}$  to  $\mathbf{Y}$ .

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$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

**X** Domain

**Y** Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y =$$

▶ Start

$\mathbf{X} \times \mathbf{Y} = \{(x, y) | x \in \mathbf{X}, y \in \mathbf{Y}\}$  : The Cartesian product  $\mathbf{X} \times \mathbf{Y}$   
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$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

**X** Domain

**Y** Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f$$



▶ Start

$\mathbf{X} \times \mathbf{Y} = \{(x, y) | x \in \mathbf{X}, y \in \mathbf{Y}\}$  : The Cartesian product  $\mathbf{X} \times \mathbf{Y}$   
 $f$  is a function from  $\mathbf{X}$  to  $\mathbf{Y}$ .

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$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

**X** Domain

**Y** Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

▶ Start

$\mathbf{X} \times \mathbf{Y} = \{(x, y) | x \in \mathbf{X}, y \in \mathbf{Y}\}$  : The Cartesian product  $\mathbf{X} \times \mathbf{Y}$   
 $f$  is a function from  $\mathbf{X}$  to  $\mathbf{Y}$ .

$$\left\{ \begin{array}{l} f \subset \mathbf{X} \times \mathbf{Y} \\ \forall x \in \mathbf{X}, \exists! y \in \mathbf{Y} \text{ s.t. } (x, y) \in f \end{array} \right\} \left\{ \begin{array}{l} \forall x \in \mathbf{X}, \exists y \in \mathbf{Y} \text{ s.t. } (x, y) \in f \\ (x, y_1) \in f \text{ and } (x, y_2) \in f \Rightarrow y_1 = y_2 \end{array} \right.$$

$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

**X** Domain

**Y** Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

▶ Start

$\mathbf{X} \times \mathbf{Y} = \{(x, y) | x \in \mathbf{X}, y \in \mathbf{Y}\}$  : The Cartesian product  $\mathbf{X} \times \mathbf{Y}$   
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$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

**X** Domain

**Y** Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

▶ Start

$\mathbf{X} \times \mathbf{Y} = \{(x, y) | x \in \mathbf{X}, y \in \mathbf{Y}\}$  : The Cartesian product  $\mathbf{X} \times \mathbf{Y}$   
 $f$  is a function from  $\mathbf{X}$  to  $\mathbf{Y}$ .

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$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

**X** Domain

**Y** Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$

▶ Start

$\mathbf{X} \times \mathbf{Y} = \{(x, y) | x \in \mathbf{X}, y \in \mathbf{Y}\}$  : The Cartesian product  $\mathbf{X} \times \mathbf{Y}$   
 $f$  is a function from  $\mathbf{X}$  to  $\mathbf{Y}$ .

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$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

**X** Domain

**Y** Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value

▶ Start

$\mathbf{X} \times \mathbf{Y} = \{(x, y) | x \in \mathbf{X}, y \in \mathbf{Y}\}$  : The Cartesian product  $\mathbf{X} \times \mathbf{Y}$   
 $f$  is a function from  $\mathbf{X}$  to  $\mathbf{Y}$ .

$$\left\{ \begin{array}{l} f \subset \mathbf{X} \times \mathbf{Y} \\ \forall x \in \mathbf{X}, \exists! y \in \mathbf{Y} \text{ s.t. } (x, y) \in f \end{array} \right\} \left\{ \begin{array}{l} \forall x \in \mathbf{X}, \exists y \in \mathbf{Y} \text{ s.t. } (x, y) \in f \\ (x, y_1) \in f \text{ and } (x, y_2) \in f \Rightarrow y_1 = y_2 \end{array} \right.$$

$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

**X** Domain

**Y** Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value of a function  $f$

▶ Start

$\mathbf{X} \times \mathbf{Y} = \{(x, y) | x \in \mathbf{X}, y \in \mathbf{Y}\}$  : The Cartesian product  $\mathbf{X} \times \mathbf{Y}$   
 $f$  is a function from  $\mathbf{X}$  to  $\mathbf{Y}$ .

$$\left\{ \begin{array}{l} f \subset \mathbf{X} \times \mathbf{Y} \\ \forall x \in \mathbf{X}, \exists! y \in \mathbf{Y} \text{ s.t. } (x, y) \in f \end{array} \right\} \left\{ \begin{array}{l} \forall x \in \mathbf{X}, \exists y \in \mathbf{Y} \text{ s.t. } (x, y) \in f \\ (x, y_1) \in f \text{ and } (x, y_2) \in f \Rightarrow y_1 = y_2 \end{array} \right.$$

$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

**X** Domain

**Y** Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value of a function  $f$  at  $x$

▶ Start

$\mathbf{X} \times \mathbf{Y} = \{(x, y) | x \in \mathbf{X}, y \in \mathbf{Y}\}$  : The Cartesian product  $\mathbf{X} \times \mathbf{Y}$   
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$$\left\{ \begin{array}{l} f \subset \mathbf{X} \times \mathbf{Y} \\ \forall x \in \mathbf{X}, \exists! y \in \mathbf{Y} \text{ s.t. } (x, y) \in f \end{array} \right\} \left\{ \begin{array}{l} \forall x \in \mathbf{X}, \exists y \in \mathbf{Y} \text{ s.t. } (x, y) \in f \\ (x, y_1) \in f \text{ and } (x, y_2) \in f \Rightarrow y_1 = y_2 \end{array} \right.$$

$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

$\mathbf{X}$  Domain

$\mathbf{Y}$  Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value of a function  $f$  at  $x$

$x$



▶ Start

$\mathbf{X} \times \mathbf{Y} = \{(x, y) | x \in \mathbf{X}, y \in \mathbf{Y}\}$  : The Cartesian product  $\mathbf{X} \times \mathbf{Y}$   
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$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

$\mathbf{X}$  Domain

$\mathbf{Y}$  Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value of a function  $f$  at  $x$

$x$  The independent variable

▶ Start

$\mathbf{X} \times \mathbf{Y} = \{(x, y) | x \in \mathbf{X}, y \in \mathbf{Y}\}$  : The Cartesian product  $\mathbf{X} \times \mathbf{Y}$   
 $f$  is a function from  $\mathbf{X}$  to  $\mathbf{Y}$ .

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$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

$\mathbf{X}$  Domain

$\mathbf{Y}$  Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value of a function  $f$  at  $x$

$x$  The independent variable

$y$

▶ Start

$\mathbf{X} \times \mathbf{Y} = \{(x, y) | x \in \mathbf{X}, y \in \mathbf{Y}\}$  : The Cartesian product  $\mathbf{X} \times \mathbf{Y}$   
 $f$  is a function from  $\mathbf{X}$  to  $\mathbf{Y}$ .

$$\left\{ \begin{array}{l} f \subset \mathbf{X} \times \mathbf{Y} \\ \forall x \in \mathbf{X}, \exists! y \in \mathbf{Y} \text{ s.t. } (x, y) \in f \end{array} \right\} \left\{ \begin{array}{l} \forall x \in \mathbf{X}, \exists y \in \mathbf{Y} \text{ s.t. } (x, y) \in f \\ (x, y_1) \in f \text{ and } (x, y_2) \in f \Rightarrow y_1 = y_2 \end{array} \right.$$

$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

$\mathbf{X}$  Domain

$\mathbf{Y}$  Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value of a function  $f$  at  $x$

$x$  The independent variable

$y$  The dependent variable

▶ Start

$\mathbf{X} \times \mathbf{Y} = \{(x, y) | x \in \mathbf{X}, y \in \mathbf{Y}\}$  : The Cartesian product  $\mathbf{X} \times \mathbf{Y}$   
 $f$  is a function from  $\mathbf{X}$  to  $\mathbf{Y}$ .

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$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

$\mathbf{X}$  Domain

$\mathbf{Y}$  Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value of a function  $f$  at  $x$

$x$  The independent variable

$y$  The dependent variable

$f(\mathbf{X})$

▶ Start

$\mathbf{X} \times \mathbf{Y} = \{(x, y) | x \in \mathbf{X}, y \in \mathbf{Y}\}$  : The Cartesian product  $\mathbf{X} \times \mathbf{Y}$   
 $f$  is a function from  $\mathbf{X}$  to  $\mathbf{Y}$ .

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$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

$\mathbf{X}$  Domain

$\mathbf{Y}$  Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value of a function  $f$  at  $x$

$x$  The independent variable

$y$  The dependent variable

$$f(\mathbf{X}) =$$

▶ Start

$\mathbf{X} \times \mathbf{Y} = \{(x, y) | x \in \mathbf{X}, y \in \mathbf{Y}\}$  : The Cartesian product  $\mathbf{X} \times \mathbf{Y}$   
 $f$  is a function from  $\mathbf{X}$  to  $\mathbf{Y}$ .

$$\left\{ \begin{array}{l} f \subset \mathbf{X} \times \mathbf{Y} \\ \forall x \in \mathbf{X}, \exists! y \in \mathbf{Y} \text{ s.t. } (x, y) \in f \end{array} \right\} \left\{ \begin{array}{l} \forall x \in \mathbf{X}, \exists y \in \mathbf{Y} \text{ s.t. } (x, y) \in f \\ (x, y_1) \in f \text{ and } (x, y_2) \in f \Rightarrow y_1 = y_2 \end{array} \right.$$

$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

**X** Domain

**Y** Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value of a function  $f$  at  $x$

$x$  The independent variable

$y$  The dependent variable

$$f(\mathbf{X}) = \{$$

▶ Start

$\mathbf{X} \times \mathbf{Y} = \{(x, y) | x \in \mathbf{X}, y \in \mathbf{Y}\}$  : The Cartesian product  $\mathbf{X} \times \mathbf{Y}$   
 $f$  is a function from  $\mathbf{X}$  to  $\mathbf{Y}$ .

$$\left\{ \begin{array}{l} f \subset \mathbf{X} \times \mathbf{Y} \\ \forall x \in \mathbf{X}, \exists! y \in \mathbf{Y} \text{ s.t. } (x, y) \in f \end{array} \right\} \left\{ \begin{array}{l} \forall x \in \mathbf{X}, \exists y \in \mathbf{Y} \text{ s.t. } (x, y) \in f \\ (x, y_1) \in f \text{ and } (x, y_2) \in f \Rightarrow y_1 = y_2 \end{array} \right.$$

$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

$\mathbf{X}$  Domain

$\mathbf{Y}$  Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value of a function  $f$  at  $x$

$x$  The independent variable

$y$  The dependent variable

$$f(\mathbf{X}) = \{y\}$$

▶ Start

$\mathbf{X} \times \mathbf{Y} = \{(x, y) | x \in \mathbf{X}, y \in \mathbf{Y}\}$  : The Cartesian product  $\mathbf{X} \times \mathbf{Y}$   
 $f$  is a function from  $\mathbf{X}$  to  $\mathbf{Y}$ .

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$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

$\mathbf{X}$  Domain

$\mathbf{Y}$  Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value of a function  $f$  at  $x$

$x$  The independent variable

$y$  The dependent variable

$$f(\mathbf{X}) = \{y\}$$



▶ Start

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Image

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Image of  $f$



▶ Start

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Image of  $f$  or

▶ Start

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Image of  $f$  or Range

▶ Start

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$x$  The independent variable

$y$  The dependent variable

$$f(\mathbf{X}) = \{y | \exists x \in \mathbf{X} \text{ s.t. } y = f(x)\} = \{f(x) | x \in \mathbf{X}\}$$

Image of  $f$  or Range of  $f$

$\mathbf{X} \times \mathbf{Y} = \{(x, y) | x \in \mathbf{X}, y \in \mathbf{Y}\}$  : The Cartesian product  $\mathbf{X} \times \mathbf{Y}$   
 $f$  is a function from  $\mathbf{X}$  to  $\mathbf{Y}$ .

$$\left\{ \begin{array}{l} f \subset \mathbf{X} \times \mathbf{Y} \\ \forall x \in \mathbf{X}, \exists! y \in \mathbf{Y} \text{ s.t. } (x, y) \in f \end{array} \right\} \left\{ \begin{array}{l} \forall x \in \mathbf{X}, \exists y \in \mathbf{Y} \text{ s.t. } (x, y) \in f \\ (x, y_1) \in f \text{ and } (x, y_2) \in f \Rightarrow y_1 = y_2 \end{array} \right.$$

$$f : \mathbf{X} \rightarrow \mathbf{Y}, \mathbf{X} \xrightarrow{f} \mathbf{Y}$$

$\mathbf{X}$  Domain

$\mathbf{Y}$  Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value of a function  $f$  at  $x$

$x$  The independent variable

$y$  The dependent variable

$$f(\mathbf{X}) = \{y | \exists x \in \mathbf{X} \text{ s.t. } y = f(x)\} = \{f(x) | x \in \mathbf{X}\}$$

Image of  $f$  or Range of  $f$